GBeam Bridge Girder Pultrusion: Section Design and Optimization

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16 Abstract

The GBeam – a fiber reinforced polymer tub-shaped bridge girder developed by the University of Maine and licensed for manufacture to AIT Bridges– has emerged as a viable replacement for traditional steel and prestressed girders in short to medium span bridges. GBeam manufacture is time and labor intensive, requiring many production steps to be done by hand, leading to low production through-put. As a possible method for accelerating production, the feasibility of producing GBeams by automated pultrusion was investigated. This involved first defining a set of geometric and material parameters to confine the design space, and then creating a series of representative designs for GBeams to be used in hypothetical, generic bridges. This led to a series of generic pultruded GBeam designs from which feasibility could be assessed. Ultimately, the limitations pultrusion places on GBeam geometry, combined with the significant initial capital investment required to begin production deemed pultrusion infeasible at this time, and prompts the search for alternative acceleration methods.

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List of Key Terms

CT: Composite tub
FRP: Fiber reinforced polymer
FVF: Fiber Volume Fraction
GBeam: FRP bridge girder developed by the U Maine, manufactured by AIT Bridges
VIP: Vacuum infusion process

Abstract

The GBeam – a fiber reinforced polymer tub-shaped bridge girder developed by the University of Maine and licensed for manufacture to AIT Bridges– has emerged as a viable replacement for traditional steel and prestressed girders in short to medium span bridges. GBeam manufacture is time and labor intensive, requiring many production steps to be done by hand, leading to low production through-put. As a possible method for accelerating production, the feasibility of producing GBeams by automated pultrusion was investigated. This involved first defining a set of geometric and material parameters to confine the design space, and then creating a series of representative designs for GBeams to be used in hypothetical, generic bridges. This led to a series of generic pultruded GBeam designs from which feasibility could be assessed. Ultimately, the limitations pultrusion places on GBeam geometry, combined with the significant initial capital investment required to begin production deemed pultrusion infeasible at this time, and prompts the search for alternative acceleration methods.

Chapter 1: Introduction

Durable, reliable infrastructure is vital for local, state, and national economic growth and development. As the economy grows, so too do the demands placed on existing infrastructure, notably on roads and bridges. To keep up with the ever-increasing demand, new, durable, rapidly erected bridges are needed. Increasingly, these bridges utilize new materials and construction techniques to improve durability, reduce construction costs, and speed erection. To respond to this need, the University of Maine developed a novel, fiber reinforced polymer (FRP) tub girder (referred to as a "GBeam") for use in new bridge construction as an alternative to conventional steel and prestressed concrete structural members (Dagher et al. 2019; Davids et al 2022a, 2022b; Davids & Schanck 2022). GBeams have been identified as a promising technology to support sustainable and durable infrastructure development, as they are comparatively light and overcome many of the challenges associated with precast NEXT beam concrete structures by reducing shipping costs and camber variability caused by prestressing. Commercialization of this technology is underway, with the evaluation of the first GBeam bridge constructed for regular traffic, the Hampden Grist Mill Bridge (HGMB) completed (Davids and Schanck 2022), two more bridges to be completed by the end of 2022, and other bridges in the design or pre-construction stage.

To date, the GBeams that have been manufactured (both for construction and research) have relied on the labor-intensive process of hand-layup and vacuum resin infusion, requiring significant manufacturing time and cost. As a potential method of streamlining the manufacturing process, the feasibility of automated manufacturing by pultrusion was suggested for investigation. To that end, this document outlines the design of GBeam girders optimized for manufacture by pultrusion. In the pultrusion process, spools of fibers and/or fabrics are bathed in thermoset resin and pulled through a die to form a specified prismatic shape. This allows for efficient, continuous manufacturing of FRP shapes, as well as the improvement of FRP fiber volume fraction (FVF) over the vacuum infusion process (VIP) – the current manufacturing process. The feasibility of GBeam pultrusion was investigated in two general steps. First, general geometric and material parameters were defined to constrain the pultruded GBeam design space. These parameters and constraints were developed through direct discussion with AIT Bridges (the current manufacturer of GBeams and prospective manufacturer of pultruded GBeams) and pultrusion experts. This then allowed representative GBeam sections to be designed, which could then be evaluated for feasibility and possible continued development.

Chapter 2: Cross-Section Definition

The first step in evaluating the feasibility of pultrusion as a GBeam manufacturing method was to narrow the possible design space with reasonable parameters and constraints on geometry. This took into consideration the physical limitations of the pultrusion process, bridge design practice, and the intention for pultruded GBeams to retain their status as a one-to-one replacement for conventional bridge girders in short to medium span bridges. In addition, the material parameters used in design were also redefined from those used in VIP design to account for the improved FVF available in pultrusion.

2.1 Geometric Properties

Since the purpose of design was to assess the feasibility of GBeams manufactured by pultrusion, sections would be designed to meet the needs of standard, generic structures rather than sections being tailored to the needs of a specific structure. The generic structure constraints were provided or assumed based on input from AIT Bridges. These included:

- Three span lengths with three girder spacings
  - 40’ span with 5’-6” spacing (with a fully cast-in-place deck)
  - 50’ span with 6’-6” spacing (with a partial precast deck)
  - 60’ span with 6’-6” spacing (with a fully precast deck)
- Girder depths between 18” and 30”
- Strength design based on Maine-modified AASHTO HL-93 loading (AASHTO 2017)
- AASHTO deflection limit of $L/1000$ to accommodate a sidewalk
- Minimum span-to-girder-depth ratio of 20

In addition to these constraints, design sections were created to optimize around two additional criteria:

1. Minimize bottom flange thickness (e.g. minimize the amount of carbon in the bottom flange)
2. Maximize span-to-depth ratio (e.g. minimize girder depth by fixing the bottom flange at the maximum value of 18 in. provided by AIT Bridges).

Finally, practical and manufacturing considerations were taken into account. Most notably, it was specified that as many web laminae as possible would be developed into both upper and lower flanges, and that voided areas in transitions between webs and flanges would be minimized to maximize the strength of transition regions. Two final profile shapes were investigated: the standard GBeam composite tub (CT) seen in Figure 1, and a double-I section seen in Figure 2.

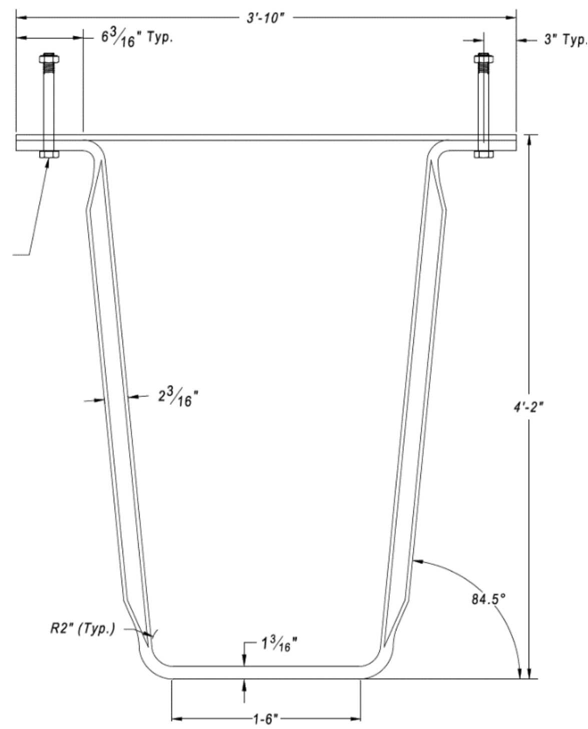

Figure 1: Potential GBeam Profile – Composite Tub

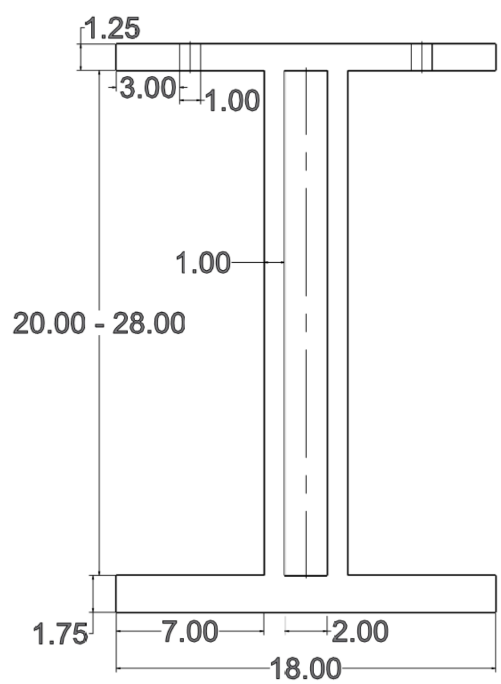

Figure 2: Potential GBeam Profile – Double-I

2.2 Material Properties

In addition to the benefits due to reduced labor, pultrusion can also have the effect of producing finished parts with higher FVF than is achievable by VIP. Parts manufactured by VIP tend to have FVFs of around 50-55%, whereas pultruded parts routinely achieve FVFs greater than 60%, improving final strength and stiffness. This can allow less material to be used in a girder with a given span than resin infused girders, positively affecting material cost and girder self-weight.

For the pultrusion process, a FVF of 62% was assumed, requiring new laminar material properties to be determined. Since material testing was not a practical option at this preliminary stage, properties of carbon and glass reinforced laminae were determined using micromechanical models, accepting their inherent inaccuracies (Barbero, 2018). These calculations are provided in Appendix A. In addition, it was assumed that laminar thickness was inversely proportional to FVF. Therefore, commonly assumed laminar thicknesses achieved through VIP were scaled by the ratio of VIP FVF to pultrusion FVF. The resulting estimated material and geometric parameters are presented in Table 1. To provide a comparison, the estimated pultruded property is given first followed by the vacuum-infused properties that have been used to-date and experimentally verified. These moduli and strengths for these laminae agree with moduli and strengths of similar laminae with similar FVF reported by others (Barbero, 2018; Miller et al., 2019; Hadigheh & Kashi, 2018). In addition, the listed values of strength were reduced by the specified statistical reliability factor of 0.85 (Tomlinson, 2013). It is important to note as well that the infused laminae use a polyester/epoxy blend matrix whereas the pultruded laminae use a polyol matrix, accounting for the drop in matrix-dominated properties of the pultruded E-glass laminae relative to their infused counterparts.

Table 1: Laminar Properties (pultruded/infused)

Property	Carbon Lamina	E-Glass Lamina
Longitudinal Elastic Modulus (ksi)	21700/14370	6630/5340
Transverse Elastic Modulus (ksi)	1090/610	1040/1620
In-Plane Poisson’s Ratio	0.238/0.280	0.250/0.280
In-Plane Shear Modulus (ksi)	690/580	690/770
Longitudinal Tensile Strength (ksi)	362/146	311/101
Lamina Thickness (in)	0.092/0.106	0.021/0.024

Chapter 3: Cross-Section Design

Once adequate constraint had been placed on the pultruded GBeam design space, generic section design could begin. Loads on the generic bridges were determined using the AASHTO LRFD Bridge Design Guide (2017) and resistances were found using the provisions of the draft FRP-CT Girder Design Manual (Schanck & Davids 2022). These calculations are provided in Appendix A. For each section, strength and flexibility calculations were performed with the aid of a series of functions written in MATLAB and provided in Appendix B. These included determining laminate properties by classical lamination theory, flexural resistance through moment curvature analysis (the "detailed method" from the FRP-CT Girder Design Manual (Schanck & Davids 2022), shear resistance by elementary strength-of-materials, and deflection by numerical integration of the curvature equation. The web foam core thickness of each section required for shear buckling resistance was determined with the FRP-CT Girder Design Manual's shear buckling nomographs that were recently developed from a comprehensive suite of 3D finite-element analyses. Finally, for the 40' spans, deck casting could cause upper flange compression failure and flange local buckling. For the former, girder non-composite strength was determined by the FRP-CT Girder Design Manual, and for the latter, the AISC Steel Construction Manual (2017) was used assuming:

- Girders acted as doubly symmetric I-sections (allowing Specification chapter F.4 to be applicable)
- Webs were not slender
- The shear ridges connecting deck and girder were not effective in resisting buckling
- Flanges were slender

The use of AISC flange local buckling provisions provided a gross approximation of double-I local stability in the absence of targeted structural testing. However, it provided an expedient solution and, as seen later, flange local buckling was not a controlling failure mode in the presented design scenarios and so does not present an apparent safety risk.

Although many of the parameters of the two shapes and separate designs differed, some were constant throughout. These were:

- Concrete compressive strength of 4 ksi
- 8" thick deck slab (partial precast deck has a 4" precast portion and 4" CIP portion)
- 18" wide, ¾" thick structural top flanges with an additional ½" of material to be machined for deck-interlock grooved connections
- 18" wide bottom flange

As previously noted, two sets of optimization criteria were used – the minimization of bottom flange thickness and the minimization of girder depth. These two criteria, along with the three pairs of bridge span and girder spacing, led to a total of six designs for each section shape – twelve designs in total. Additionally, in all cases a maximum amount of web material was kept continuous through the entirety of the section to develop web strength and provide continuity between, and confinement of the various section portions.

3.1 Composite Tub Girders

Per previous CT design experience, and in the absence of more accurate predictions of load distribution, the CT girders were designed using AASHTO distribution factors for concrete spread box girders (using Table 4.6.2.2.2b-1 and 4.6.2.2.3a-1). It is important to note that since the specified limits of applicability listed by AASHTO were obeyed, the distribution factors for the 40' span were determined by the lever rule.

For each of the designs, as much of the glass laminae in the webs was made continuous through the section as possible, filling in the available space not already taken by the carbon required for flexural strength. Any additional remaining void space was filled using unidirectional glass rovings. Table 2 describes the geometries of the designs, Table 3 presents the ratios of design strength to demand, and Figures 3-8 show drawings of the design cross-sections. Note that these figures also include ½" shear ridges cast into the overlying concrete/grout to provide composite action. As expected, the sections optimized for bottom flange thickness tended to be more materially efficient, whereas the sections optimized for girder depth tended to enclose a smaller volume. It can also be seen from the last column of Table 2 that a comparatively small amount of web glass is able to be developed into the bottom flange for the sections optimized for smaller depth. This brings their ability to achieve their theoretical strength into question and suggests that this type of optimization may not lead to viable designs.

Table 2: Summary of CT Designs

Parameter Minimized	Span (ft)	Depth (in)	Bottom Flange Thickness (in)	Web Thickness (in)	Core Thickness (in)	Percentage of Continuous Web Laminae (%)
Bottom Flange Thickness	40	20	1.41	1.21	0.5	100
	50	26	1.64	1.21	0.5	100
	60	30	1.74	1.21	0.5	76
Girder Depth	40	18	1.74	1.30	0.5	79
	50	22	1.73	1.26	0.5	56
	60	28	1.75	1.26	0.5	11

Table 3: CT Designs Capacity-Demand Ratios

Parameter Minimized	Span (ft)	Depth (in)	$\frac{\phi M_n}{M_u}$	$\frac{\phi V_n}{V_u}$	$\frac{\Delta}{\Delta_{max}}$
Bottom Flange Thickness	40	20	2.50	1.04	0.981
	50	26	3.02	1.17	0.983
	60	30	2.74	1.07	1.04
Girder Depth	40	18	2.31	1.00	0.933
	50	22	2.64	1.01	0.980
	60	28	2.59	1.00	0.954

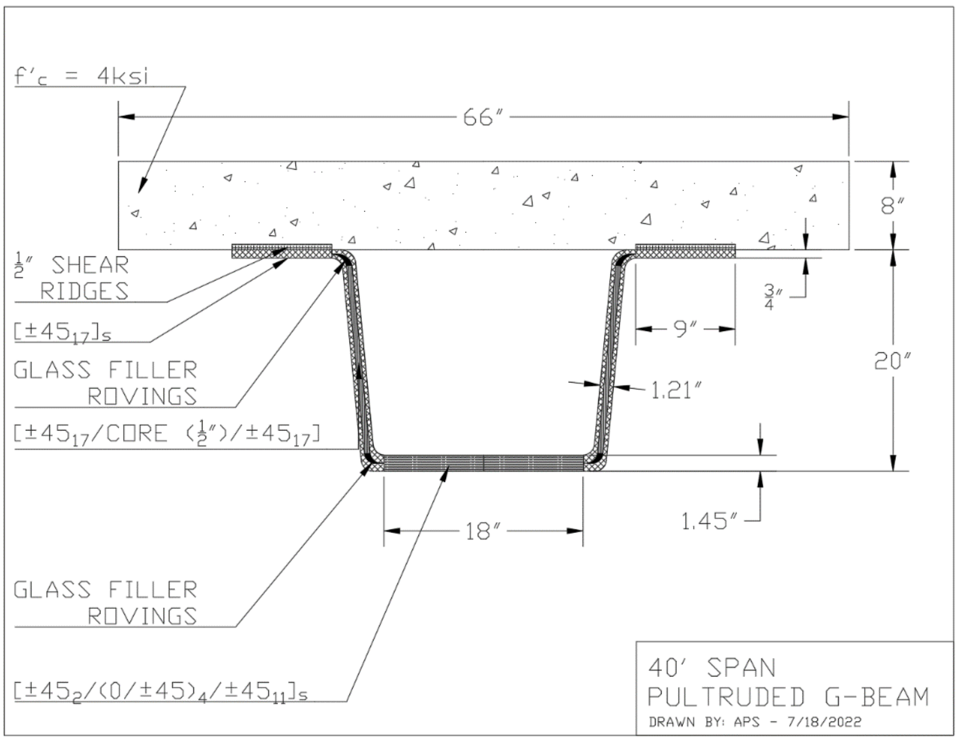

Figure 3: CT Girder – 40' Span – Minimum Bottom Flange Thickness

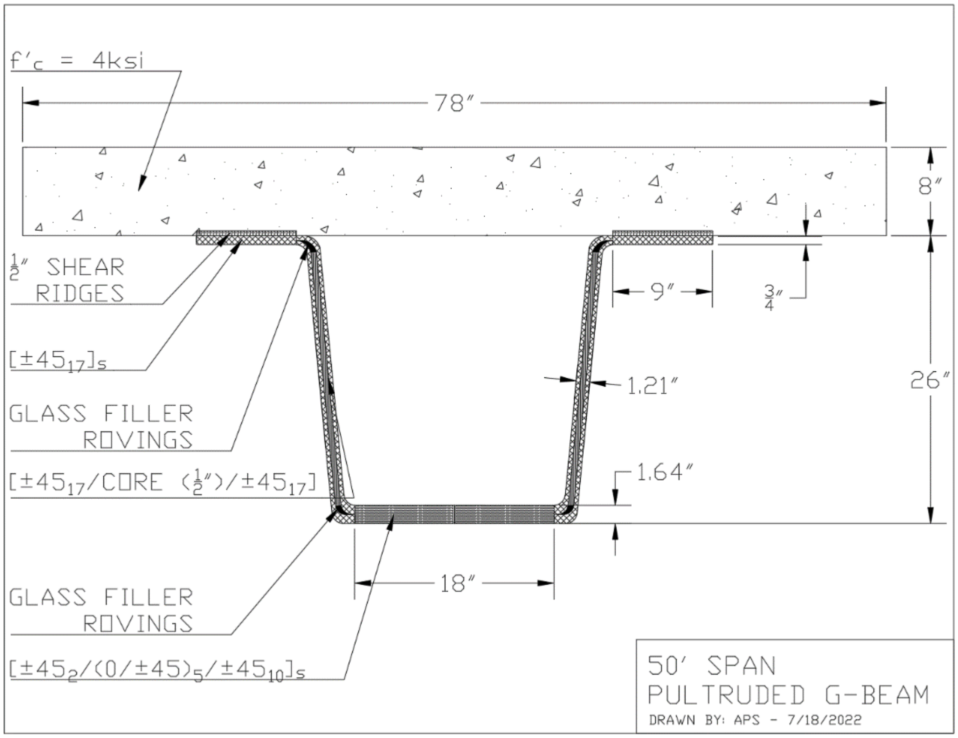

Figure 4: CT Girder – 50' Span – Minimum Bottom Flange Thickness

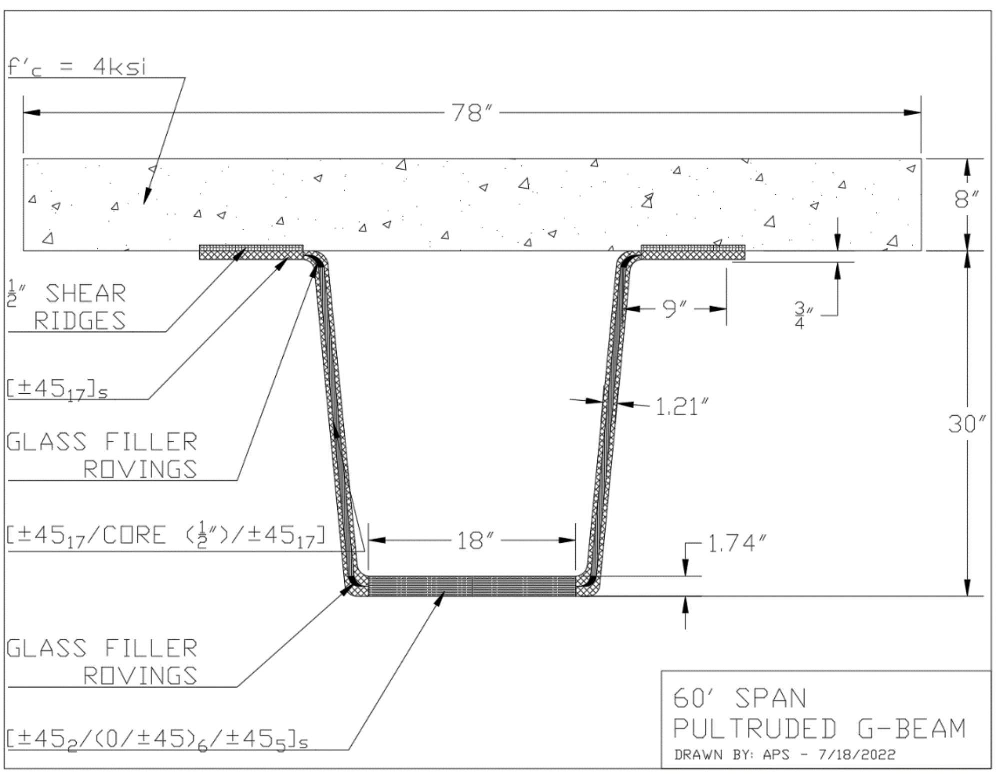

Figure 5: CT Girder – 60' Span – Minimum Bottom Flange Thickness

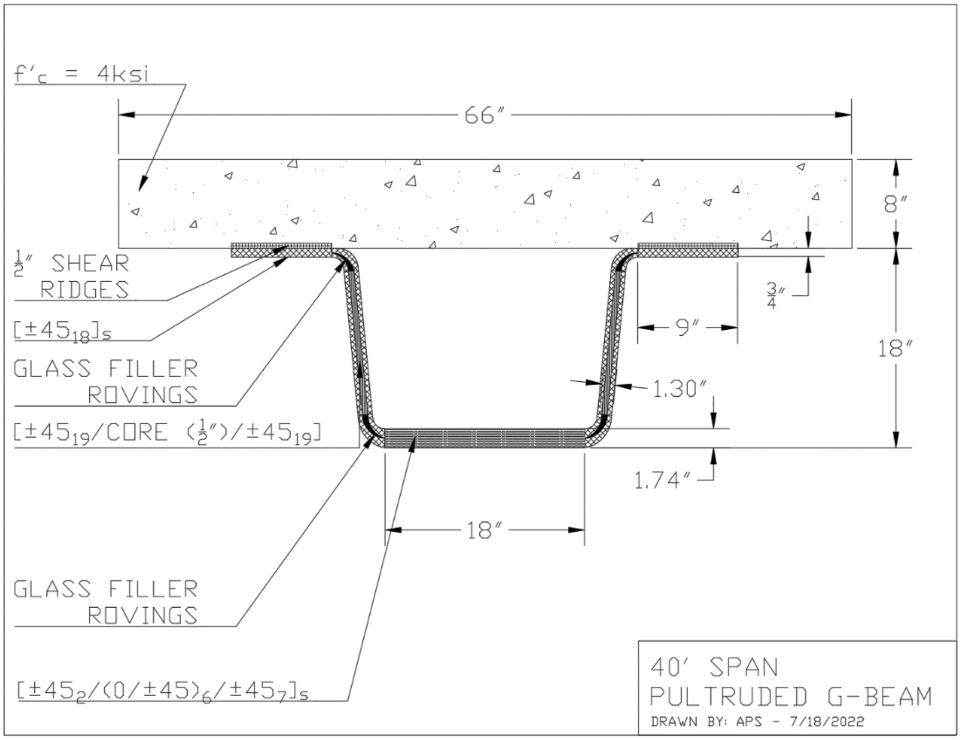

Figure 6: CT Girder – 40' Span – Minimum Depth

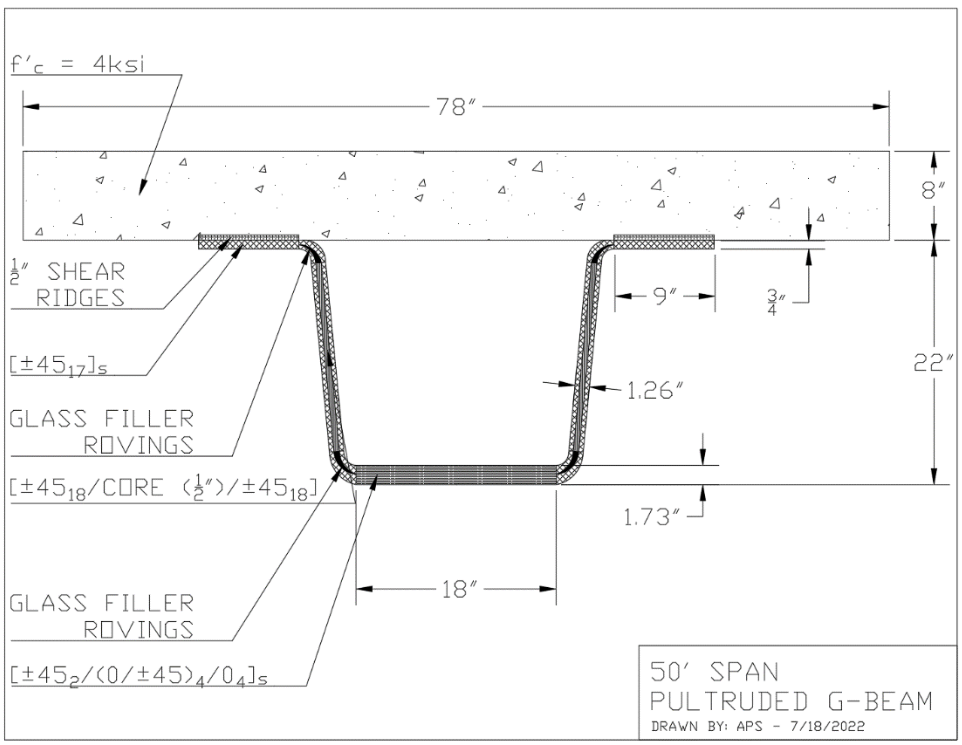

Figure 7: CT Girder – 50' Span – Minimum Depth

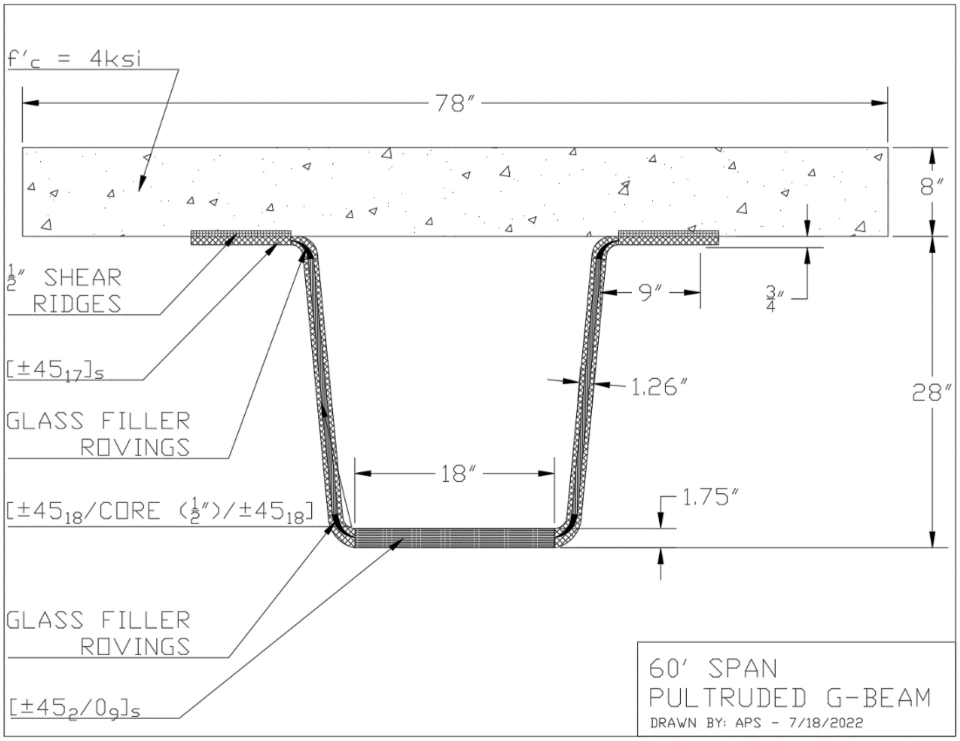

Figure 8: CT Girder – 60' Span – Minimum Depth

3.2 Double-I Girders

In the absence of more accurate predictions of load distribution, the double-I girders were designed using AASHTO distribution factors for steel I-girders (using Table 4.6.2.2.2b-1 and 4.6.2.2.3a-1 in AASHTO). The third term in the moment distribution factor equations (the term containing the factor considering relative stiffness between deck and girder) was taken as a constant 1.02 per Table 4.6.2.2.1-2.

For each of the designs, as much of the glass laminae in the webs was made continuous through the section as possible. However, due to the way these sections would be manufactured, relatively few web face sheet laminae could be continuous, and in no case were the face sheet laminae continuous through the entire section. Instead of relying on this continuity, shear transfer would be provided solely by shear planes between layers of carbon and multiple layers of glass. Table 4 describes the geometries of the designs, and Table 5 presents the ratios of design strength to demand. Figures 9-14 show drawings of the design cross-sections. Note that these figures also include ½" shear ridges cast into the overlying concrete/grout to provide composite action. Again, as seen in the final column of Table 3, the small amount of glass developed into the bottom flange brings these section's theoretical capacity into question, especially for the sections designed to optimize girder depth. This suggests that the double-I section shape may not be viable in any configuration.

Table 4: Summary of Double-I Designs

Minimization	Span (ft)	Depth (in)	Bottom Flange Thickness (in)	Web Thickness (in)	Core Thickness (in)	Percentage of Continuous Web Laminae (%)
Bottom Flange Thickness	40	20	1.41	1.21	0.5	50
	50	26	1.64	1.21	0.5	50
	60	30	1.74	1.21	0.5	44
Girder Depth	40	18	1.75	1.30	0.5	50
	50	22	1.74	1.26	0.5	11
	60	28	1.75	1.34	0.5	0

Table 5: Double-I Designs Capacity-Demand Ratios

Parameter Minimized	Span (ft)	Depth (in)	$\frac{\phi M_n}{M_u}$	$\frac{\phi V_n}{V_u}$	$\frac{\Delta}{\Delta_{max}}$
Bottom Flange Thickness	40	20	2.94	1.04	0.979
	50	26	2.98	1.17	0.953
	60	30	3.30	1.01	1.04
Girder Depth	40	18	2.77	1.01	0.933
	50	22	2.61	1.01	0.980
	60	28	2.56	1.05	0.954

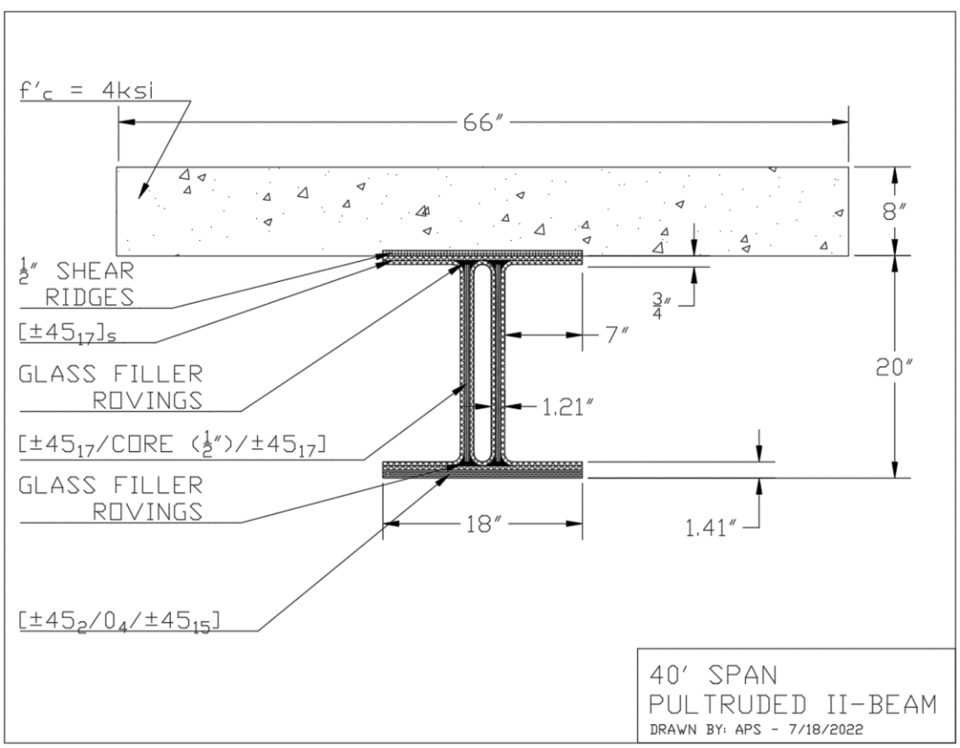

Figure 9: Double-I Girder – 40' Span – Minimum Bottom Flange Thickness

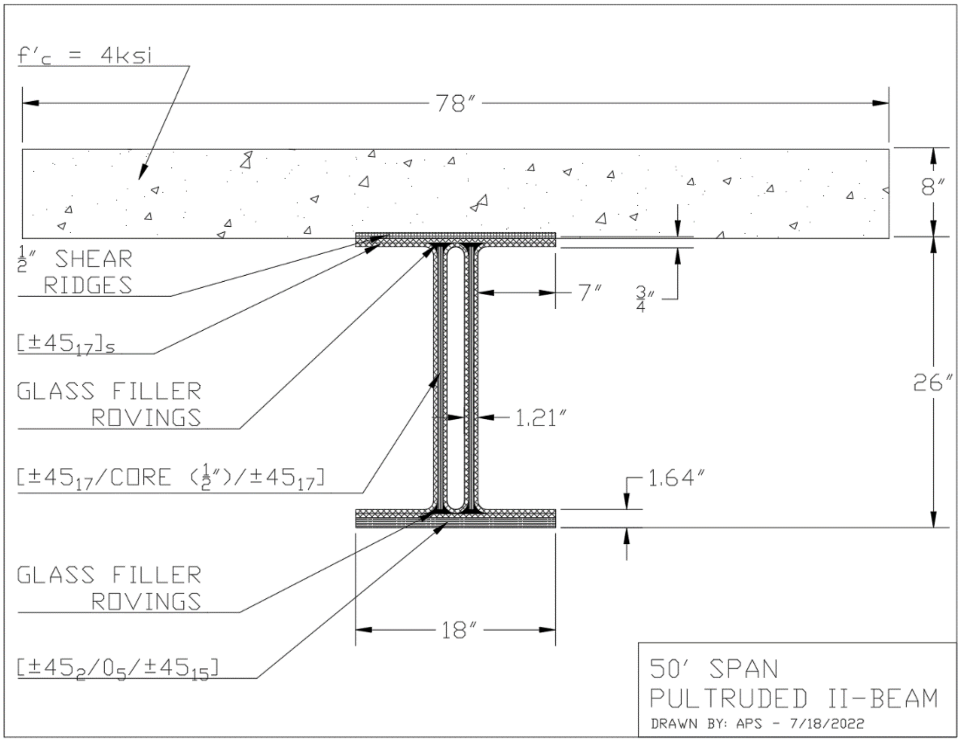

Figure 10: Double-I Girder – 50' Span – Minimum Bottom Flange Thickness

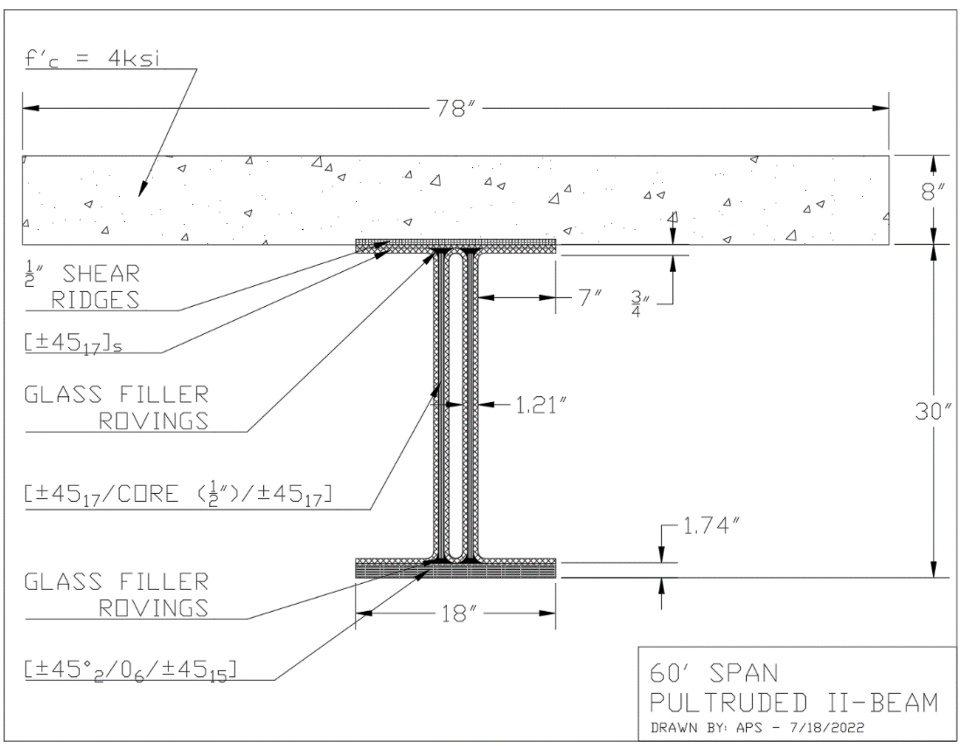

Figure 11: Double-I Girder – 60' Span – Minimum Bottom Flange Thickness

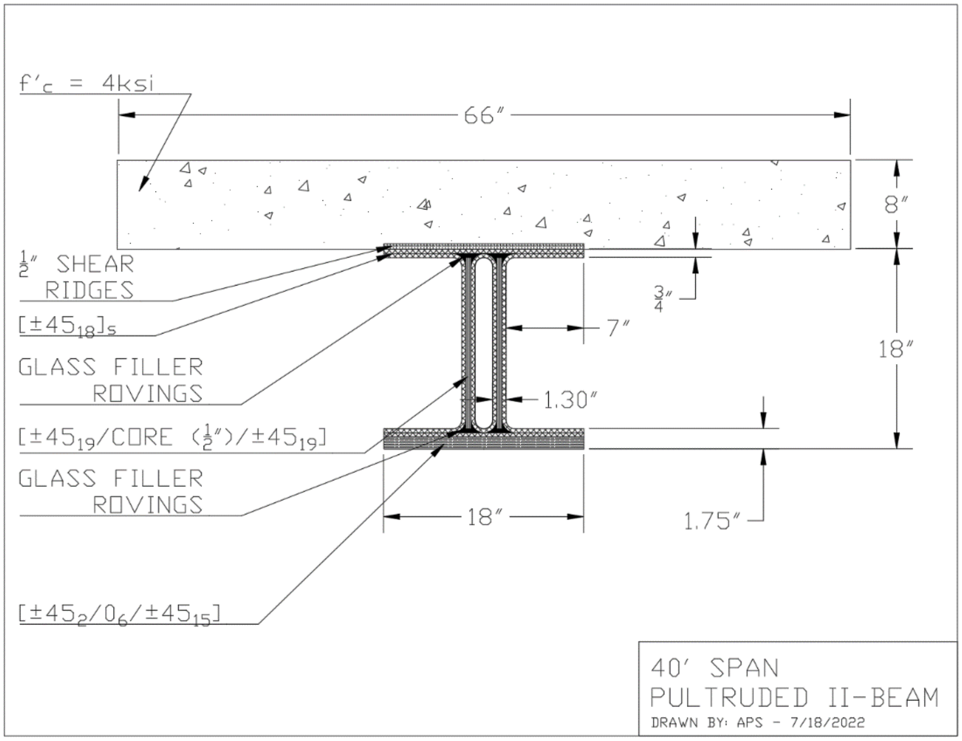

Figure 12: Double-I Girder – 40' Span – Minimum Depth

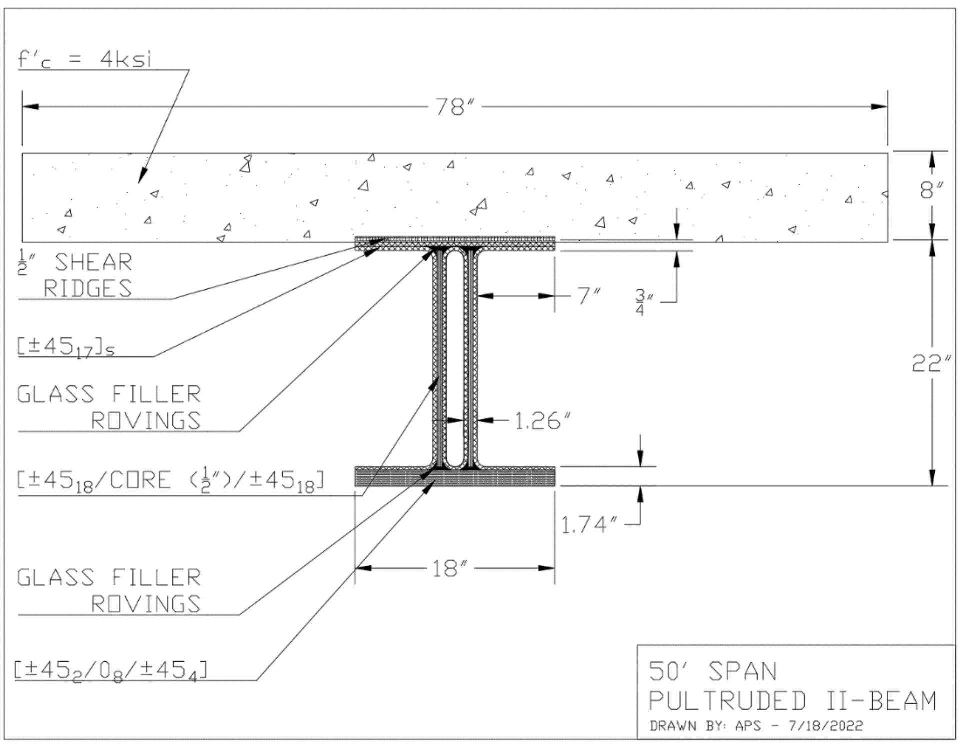

Figure 13: Double-I Girder – 50' Span – Minimum Depth

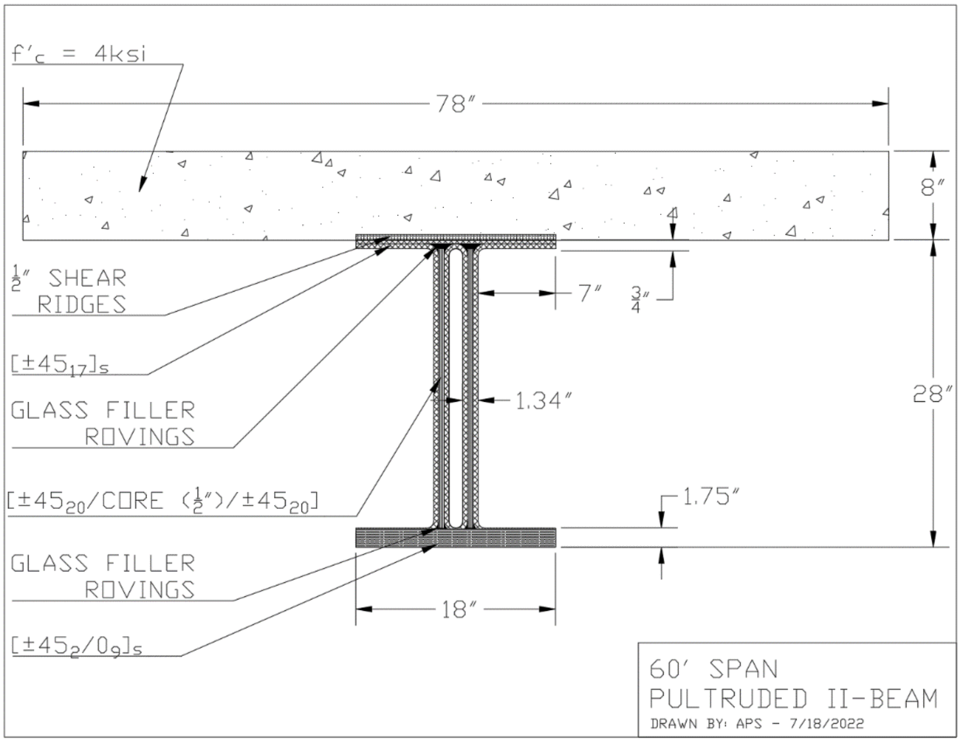

Figure 14: Double-I Girder – 60' Span – Minimum Depth

Chapter 4: Results and Discussion

As shown in the previous section, the CT sections yield designs capable of providing for the strength and serviceability needs of the given structures. However, it must be noted that the double-I girder concept has drawbacks from which the CT girder does not suffer. These include the fact that it relies on single shear planes to attach carbon and glass laminae in the bottom flange (the CT girder allows the web glass lamina to be layered with the additional flange layers), and that its transition radius zones between webs and flanges (especially for longer spans) contain very little reinforcement. These factors may prevent the double-I section from having sufficient capacity. The central cavity between the webs of the double-I section may also be difficult to form during manufacturing. These factors suggest that the double-I concept may not be viable.

For both the CT and double-I girders, the sections optimized for bottom flange thickness tended to be more materially efficient, whereas the sections optimized for girder depth tended to enclose a smaller volume. However, as seen in both Tables 3 and 5, neither shape provides an efficient use of flexural reinforcement. As can be seen, the factored moment resistance of each design exceeds the factored moment demand by between 230 and 330%, whereas the deflection criterion is met closely. This suggests that the girders' flexural stiffness requirements vastly outweigh their strength requirements, leading to inherent inefficiency.

In addition to the inefficiencies in usage of flexural reinforcement, pultrusion of GBeams introduces inherent inefficiency as a result of the necessity of retaining a prismatic section. In GBeams manufactured by VIP, the layup of portions of the cross section can be varied along the span to remove unused reinforcement. For instance, material in the webs can be removed near midspan where shear is at a minimum, and material in the bottom flange can be removed near the ends where moments are smaller. This is not possible with pultrusion, where the material feed system and rigid tooling does not easily accommodate changes in layup, leading to poor material usage.

Chapter 5: Conclusions and Future Work

In this study pultrusion was investigated as a possible method to reduce the labor and time costs associated with manufacturing GBeams by VIP. This required a narrowing of the design space and creating preliminary designs for girder sections applicable to a set of generic bridge models. Each of the developed designs was able to provide adequate flexural strength, shear strength, and flexural stiffness based on design loading. However, the required continuity between areas of the cross section provided by continuous web material was provided only by the CT girders that had been designed to minimize bottom flange thickness, which are therefore the only viable cross-section considered here.

Despite a viable design having been attained, the feasibility of this design is still in question. While increasing product through-put over VIP, pultrusion makes some other important geometric considerations such as camber much more difficult. Finally, creation of any pultrusion manufacturing line requires a significant capital investment. The fact that the proposed GBeams are large and complex compared with other pultruded sections will increase this investment. For all these reasons it was determined that pultruding GBeams is not a feasible method of reducing labor and time costs at this time.

Although pultrusion proved infeasible, other methods of decreasing labor and time costs may prove effective. For instance, a promising possibility is the partial automation of the fabric lay-up process. Fabric layup – the act of cutting fabric from the roll, placing it in a form, and tacking it in place – tends to require the most time and effort of any process in VIP manufacturing. Accelerating this process by automating any or all the given steps could serve to positively affect through-put speed and decrease manufacturing costs. This could also retain the benefits of VIP including the straightforward inclusion of camber and curvature, as well as non-prismatic layups. Therefore, in order to accelerate GBeam manufacture and reduce labor costs, layup automation should be investigated.

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Appendix A: Design Calculations

G-Beam Pultrusion Section Design

APS - 7/22

Design Parameters

3 Combinations of Span Length and Girder Spacing:

40' Span, 5'-6" Spacing (CIP Deck)
50' Span, 6'-6" Spacing (Partial PC Deck)
60' Span, 6'-6" Spacing (Full PC Deck)

2 Potential Sectional Shapes:

Standard Tub
Double I / Roman II

2 Main Variables to Minimize in Design - Optimized Separately:

Bottom Flange Thickness (e.g. Amount of Carbon in Bottom Flange)
Girder Depth

Design Based on:

AASHTO LRFD Bridge Design Manual
Draft FRP Girder Design Guide

Note: Strength calculations performed with a separate MATLAB code which analyzes a given section for moment capacity by moment-curvature analysis, shear capacity by fundamental strength of materials, and deflection by numerically solving the differential equation of curvature. This code is provided separately.

Approximate Laminate Properties

Due to differing material properties of the matrix and the higher fiber volume fraction (FVF) available from pultrusion, new laminar properties must be determined

Assumptions:
- Laminar thickness scales with FVF
- Analyitical/emperical equations are valid

Fiber Volume Fraction

$V := 0.62$

Carbon:

$E_C := 34800\ ksi$

$F_C := 580\ ksi$

$\nu_C := 0.2$

Glass

$E_G := 10440\ ksi$

$F_G := 490\ ksi$

$\nu_G := 0.22$

Polyol Polymer

$E_M := 421\ ksi$

$\nu_M := 0.3$

$$G_M := \frac{E_M}{2 \cdot (1+\nu_M)} = 161.923\ ksi$$

Carbon Composite

$$E_{1C} := E_C \cdot V + E_M \cdot (1-V) = \left(2.174 \cdot 10^4\right)\ ksi$$

$$E_{2C} := \left(\frac{V}{E_C} + \frac{1-V}{E_M}\right)^{-1} = \left(1.086 \cdot 10^3\right)\ ksi$$

$$\nu_{12C} := \nu_C \cdot V + \nu_M \cdot (1-V) = 0.238$$

$$G_C := G_M \cdot \frac{1+V}{1-V} = 690.304\ ksi$$

$$F_{1tC} := F_C \cdot \left(V + \frac{E_M}{E_C} \cdot (1-V)\right) = 362.266\ ksi$$

$$t_C := 0.106\ in \cdot \frac{0.54}{V} = 0.092\ in$$

Glass Composite

$$E_{1G} := E_G \cdot V + E_M \cdot (1-V) = \left(6.633 \cdot 10^3\right)\ ksi$$

$$E_{2G} := \left(\frac{V}{E_G} + \frac{1-V}{E_M}\right)^{-1} = \left(1.04 \cdot 10^3\right)\ ksi$$

$$\nu_{12G} := \nu_G \cdot V + \nu_M \cdot (1-V) = 0.25$$

$$G_G := G_M \cdot \frac{1+V}{1-V} = 690.304\ ksi$$

$$F_{1tG} := F_G \cdot \left(V + \frac{E_M}{E_G} \cdot (1-V)\right) = 311.309\ ksi$$

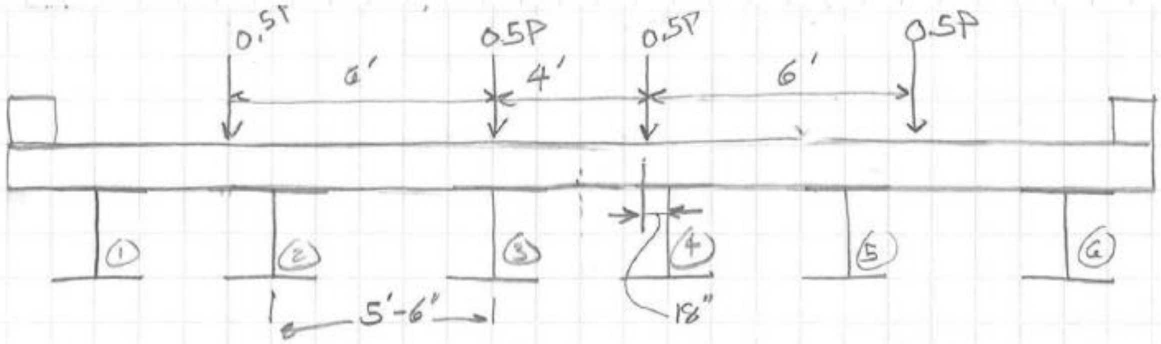
$$t_G := 0.024\ in \cdot \frac{0.54}{V} = 0.021\ in$$

$$\varepsilon_{1c} := 0.012$$

Tub Girder Design

Distribution Factors - Use AASHTO Box Beam DFs

40' Span: AASHTO DF Not applicable (S < 6') - Use Lever Rule



$R := 0.5 + 0.5 \cdot \frac{18}{66} = 0.636$ $\quad g_{40} := R = 0.636$ $\quad g_{v40} := g_{40} = 0.636$

50' Span: Assume d = 30"

$S := 6.5\ ft \quad d := 30\ in \quad L := 50\ ft$

Moment

$$g_1 := \left(\frac{S}{3\ ft}\right)^{0.35} \cdot \left(\frac{S \cdot d}{L^2}\right)^{0.25} = 0.372$$

$$g_2 := \left(\frac{S}{6.3\ ft}\right)^{0.6} \cdot \left(\frac{S \cdot d}{L^2}\right)^{0.125} = 0.543$$

$$g_{50} := \max\left(g_1, g_2\right) = 0.543$$

Shear

$$g_1 := \left(\frac{S}{10\ ft}\right)^{0.6} \cdot \left(\frac{d}{L}\right)^{0.1} = 0.572$$

$$g_2 := \left(\frac{S}{7.4\ ft}\right)^{0.8} \cdot \left(\frac{d}{L}\right)^{0.1} = 0.668$$

$$g_{v50} := \max\left(g_1, g_2\right) = 0.668$$

60' Span: Assume d = 30"

$S := 6.5\ ft \quad d := 30\ in \quad L := 60\ ft$

Moment

$$g_1 := \left(\frac{S}{3\ ft}\right)^{0.35} \cdot \left(\frac{S \cdot d}{L^2}\right)^{0.25} = 0.34$$

$$g_2 := \left(\frac{S}{6.3\ ft}\right)^{0.6} \cdot \left(\frac{S \cdot d}{L^2}\right)^{0.125} = 0.519$$

$$g_{60} := \max\left(g_1, g_2\right) = 0.519$$

Shear

$$g_1 := \left(\frac{S}{10\ ft}\right)^{0.6} \cdot \left(\frac{d}{L}\right)^{0.1} = 0.562$$

$$g_2 := \left(\frac{S}{7.4\ ft}\right)^{0.8} \cdot \left(\frac{d}{L}\right)^{0.1} = 0.656$$

$$g_{v60} := \max\left(g_1, g_2\right) = 0.656$$

Loads

Girder Dead Load

$A_{frp} := 18\ in \cdot 1.75\ in + 4\ in \cdot 28\ in + 18\ in \cdot 1.25\ in = 1.153\ ft^2$

$w_{frp} := 0.1156\ \frac{kip}{ft^3} \cdot A_{frp} = 0.133\ \frac{kip}{ft}$

40' Span $L := 40\ ft$

Dead Loads

$w_{deck} := 0.15\ \frac{kip}{ft^3} \cdot 5.5\ ft \cdot 8\ in = 0.55\ \frac{kip}{ft}$

$w_{ws} := 0.145\ \frac{kip}{ft^3} \cdot 5.5\ ft \cdot 3\ in = 0.199\ \frac{kip}{ft}$

$w_{DC} := w_{deck} + w_{frp} = 0.683\ \frac{kip}{ft}$

$M_{DC40} := \frac{w_{DC} \cdot L^2}{8} = 136.652\ ft \cdot kip$ $\qquad V_{DC40} := w_{DC} \cdot \frac{L}{2} = 13.665\ kip$

$M_{DW40} := \frac{w_{ws} \cdot L^2}{8} = 39.875\ ft \cdot kip$ $\qquad V_{DW40} := w_{ws} \cdot \frac{L}{2} = 3.988\ kip$

Live Load

$M := \left(1.33 \cdot 32\ kip \cdot \left(\frac{L}{4} + \frac{0.5 \cdot L - 14\ ft}{L} \cdot \frac{L}{2} \cdot \left(1 + \frac{8}{32}\right)\right) + \frac{.64 \cdot kip}{ft} \cdot \frac{L^2}{8}\right) \cdot 1.25 = 891.5\ ft \cdot kip$

$V := \left(1.33 \cdot 32\ kip \cdot \left(1 + \frac{L - 14\ ft}{L} \cdot \left(1 + \frac{8}{32}\right)\right) + 0.64\ \frac{kip}{ft} \cdot \frac{L}{2}\right) \cdot 1.25 = 112.425\ kip$

$M_{LL40} := M \cdot g_{40} = 567.318\ ft \cdot kip$

$V_{LL40} := V \cdot g_{v40} = 71.543\ kip$

$M_{u40} := 1.75 \cdot M_{LL40} + 1.5 \cdot M_{DW40} = \left(1.053 \cdot 10^3\right)\ ft \cdot kip$

$\gamma M_{DC40} := 1.25 \cdot M_{DC40} = 170.815\ (ft \cdot kip)$

$V_{u40} := 1.75 \cdot V_{LL40} + 1.25 \cdot V_{DC40} + 1.5 \cdot V_{DW40} = 148.263\ kip$

$\Delta_{40} := \frac{L}{1000} = 0.48\ in$

50' Span $L := 50\ ft$

Dead Loads

$$w_{deck} := 0.15\ \frac{kip}{ft^3} \cdot 6.5\ ft \cdot 8\ in = 0.65\ \frac{kip}{ft}$$

$$w_{ws} := 0.145\ \frac{kip}{ft^3} \cdot 6.5\ ft \cdot 3\ in = 0.236\ \frac{kip}{ft}$$

$$w_{DC} := w_{deck} + w_{frp} = 0.783\ \frac{kip}{ft}$$

$$M_{DC50} := \frac{w_{DC} \cdot L^2}{8} = 244.769\ ft \cdot kip$$

$$M_{DW50} := \frac{w_{ws} \cdot L^2}{8} = 73.633\ ft \cdot kip$$

$$V_{DC50} := w_{DC} \cdot \frac{L}{2} = 19.582\ kip$$

$$V_{DW50} := w_{ws} \cdot \frac{L}{2} = 5.891\ kip$$

Live Loads

$$M := \left(1.33 \cdot 32\ kip \cdot \left(\frac{L}{4} + \frac{0.5 \cdot L - 14\ ft}{L} \cdot \frac{L}{2} \cdot \left(1 + \frac{8}{32}\right)\right) + \frac{.64 \cdot kip}{ft} \cdot \frac{L^2}{8}\right) \cdot 1.25 = \left(1.281 \cdot 10^3\right)\ ft \cdot kip$$

$$V := \left(1.33 \cdot 32\ kip \cdot \left(1 + \frac{L - 14\ ft}{L} \cdot \left(1 + \frac{8}{32}\right)\right) + 0.64\ \frac{kip}{ft} \cdot \frac{L}{2}\right) \cdot 1.25 = 121.08\ kip$$

$$M_{LL50} := M \cdot g_{50} = 695.38\ ft \cdot kip$$

$$V_{LL50} := V \cdot g_{v50} = 80.894\ kip$$

$$M_{u50} := 1.75 \cdot M_{LL50} + 1.5 \cdot M_{DW50} = \left(1.327 \cdot 10^3\right)\ ft \cdot kip$$

$$\gamma M_{DC50} := 1.25 \cdot M_{DC50} = 305.961\ (ft \cdot kip)$$

$$V_{u50} := 1.75 \cdot V_{LL40} + 1.25 \cdot V_{DC50} + 1.5 \cdot V_{DW50} = 158.513\ kip$$

$$\Delta_{50} := \frac{L}{1000} = 0.6\ in$$

60' Span $L := 60\ ft$

Dead Loads

$w_{deck} := 0.15\ \frac{kip}{ft^3} \cdot 6.5\ ft \cdot 8\ in = 0.65\ \frac{kip}{ft}$

$w_{ws} := 0.145\ \frac{kip}{ft^3} \cdot 6.5\ ft \cdot 3\ in = 0.236\ \frac{kip}{ft}$

$w_{DC} := w_{deck} + w_{frp} = 0.783\ \frac{kip}{ft}$

$M_{DC60} := \frac{w_{DC} \cdot L^2}{8} = 352.468\ ft \cdot kip$

$M_{DW60} := \frac{w_{ws} \cdot L^2}{8} = 106.031\ ft \cdot kip$

$V_{DC60} := w_{DC} \cdot \frac{L}{2} = 23.498\ kip$

$V_{DW60} := w_{ws} \cdot \frac{L}{2} = 7.069\ kip$

Live Loads

$M := \left(1.33 \cdot 32\ kip \cdot \left(\frac{L}{4} + \frac{0.5 \cdot L - 14\ ft}{L} \cdot \frac{L}{2} \cdot \left(1 + \frac{8}{32}\right)\right) + \frac{.64 \cdot kip}{ft} \cdot \frac{L^2}{8}\right) \cdot 1.25 = \left(1.69 \cdot 10^3\right)\ ft \cdot kip$

$V := \left(1.33 \cdot 32\ kip \cdot \left(1 + \frac{L - 14\ ft}{L} \cdot \left(1 + \frac{8}{32}\right)\right) + 0.64\ \frac{kip}{ft} \cdot \frac{L}{2}\right) \cdot 1.25 = 128.183\ kip$

$M_{LL60} := M \cdot g_{60} = 876.696\ ft \cdot kip$

$V_{LL60} := V \cdot g_{v60} = 84.092\ kip$

$M_{u60} := 1.75 \cdot M_{LL60} + 1.5 \cdot M_{DW60} = \left(1.693 \cdot 10^3\right)\ ft \cdot kip$

$\gamma M_{DC60} := 1.25 \cdot M_{DC60} = 440.584\ (ft \cdot kip)$

$V_{u60} := 1.75 \cdot V_{LL60} + 1.25 \cdot V_{DC60} + 1.5 \cdot V_{DW60} = 187.137\ kip$

$\Delta_{60} := \frac{L}{1000} = 0.72\ in$

Design Checks

Minimum Bottom Flange Thickness
40' Span

$\phi M_n := 2600\ ft \cdot kip$ $\quad$ if $(\phi M_n \geq M_{u40}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

$\phi V_n := 154\ kip$ $\quad$ if $(\phi V_n \geq V_{u40}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

$\Delta := 0.471\ in$ $\quad$ if $(\Delta \leq \Delta_{40}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.357\ in \quad h_w := 17.83\ in \quad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 11.879\ \frac{kip}{in}$$

Results from 24" and 36" nomographs

$r_1 := 0.33 \quad h_1 := 24\ in$

$r_2 := 0.33 \quad h_2 := 36\ in$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot (r_2 - r_1) = 0.33 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.352\ in$$

Say:
$t_{core} := 0.5\ in$

40' span uses a CIP deck. Prior to curing, girder is subject to top-flange compression failure and Flange Local Buckling

Top Flange Compression

$E_{tf} := 2069\ ksi \quad n_{tf} := 0.167 \quad b_{tf} := 18\ in \quad t_{tf} := 0.714\ in \quad S_x := 53.61\ in^3$

$I_{NC} := 923.36\ in^4 \quad d := 20\ in \quad y_{NC} := 2.78\ in$

$$M_{PC} := \frac{E_{tf} \cdot \varepsilon_{1c} \cdot I_{NC}}{n_{tf} \cdot (d - y_{NC})} = 664.327\ ft \cdot kip$$

$\phi M_{PC} := 0.75 \cdot M_{PC} = 498.245\ ft \cdot kip$ $\quad$ if $(\phi M_{PC} \geq \gamma M_{DC40}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

Flange Local Buckling

Assume:
- Girder acts as a doubly-symetric I-section such that AISC F.4 is Applicable
- Web is not slender
- Shear Ridges are not effective in resisting FLB
- Flanges are slender

$$k_c := \frac{4}{\sqrt{\dfrac{h_w}{2 \cdot t_{fs} + t_{core}}}} = 1.044 \qquad k_c := \mathbf{if}\left(k_c > 0.76, 0.76, k_c\right) = 0.76$$

$$\lambda := \frac{b_{tf}}{2 \cdot t_{tf}} = 12.605$$

$$M_{FLB} := \frac{0.9 \cdot E_{tf} \cdot k_c \cdot S_x}{n_{tf} \cdot \lambda^2} = 238.274\ (ft \cdot kip)$$

$$\phi M_{FLB} := 0.75 \cdot M_{FLB} = 178.705\ ft \cdot kip \qquad \mathbf{if}\left(\phi M_{FLB} \geq \gamma M_{DC40}, \text{“OK”}, \text{“NG”}\right) = \text{“OK”}$$

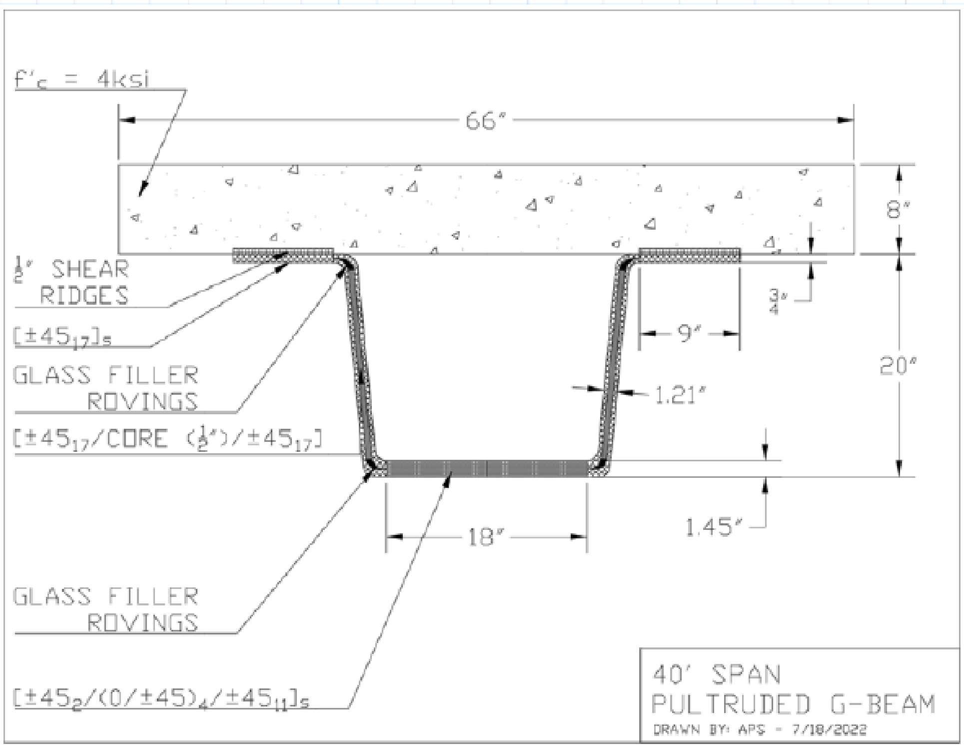
Resulting Section:

Depth: 20"

Bottom Flange:
18" Wide
1.45" Thick

Webs:
1.21" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



50' Span

$\phi M_n := 4010\ ft \cdot kip$ $\qquad$ $\text{if}\left(\phi M_n \ge M_{u50}, \text{“OK”}, \text{“NG”}\right) = \text{“OK”}$

$\phi V_n := 185\ kip$ $\qquad$ $\text{if}\left(\phi V_n \ge V_{u50}, \text{“OK”}, \text{“NG”}\right) = \text{“OK”}$

$\Delta := 0.590\ in$ $\qquad$ $\text{if}\left(\Delta \le \Delta_{50}, \text{“OK”}, \text{“NG”}\right) = \text{“OK”}$

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.357\ in \qquad h_w := 23.65\ in \qquad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 8.956\ \frac{kip}{in}$$

Results from 24" and 36" nomographs

$r_1 := .2 \qquad h_1 := 24\ in$

$r_2 := .2 \qquad h_2 := 36\ in$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot \left(r_2 - r_1\right) = 0.2 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.179\ in$$

Say, for standardization:
$t_{core} := 0.5\ in$

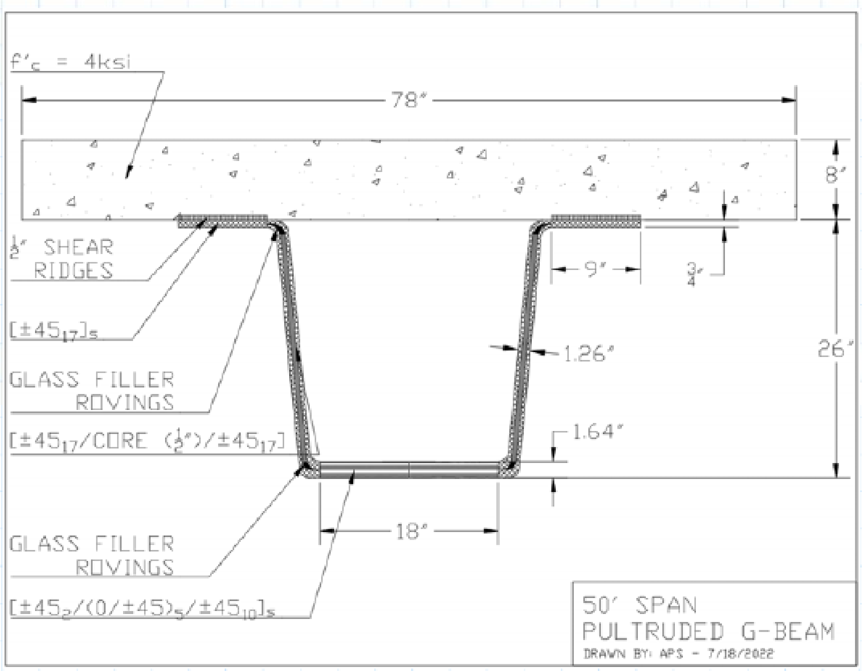
Resulting Section:

Depth 26"

Bottom Flange:
18" Wide
1.64" Thick

Webs:
0.21" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



60' Span

$\phi M_n := 4640\ ft \cdot kip$ $\qquad$ $\text{if}\left(\phi M_n \ge M_{u60}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\phi V_n := 200\ kip$ $\qquad$ $\text{if}\left(\phi V_n \ge V_{u60}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\Delta := 0.7512\ in$ $\qquad$ $\text{if}\left(\Delta \le \Delta_{60}, \text{"OK"}, \text{"NG"}\right) = \text{"NG"}$

Section does not meet deflection criteria, but only by a small amount. Call it OK.

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.357\ in$ $\qquad$ $h_w := 27.54\ in$ $\qquad$ $\phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 7.691\ \frac{kip}{in}$$

Results from 24" and 36" nomographs

$r_1 := .2$ $\qquad$ $h_1 := 24\ in$

$r_2 := .35$ $\qquad$ $h_2 := 36\ in$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot \left(r_2 - r_1\right) = 0.244 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.231\ in$$

Say:

$t_{core} := 0.25\ in$

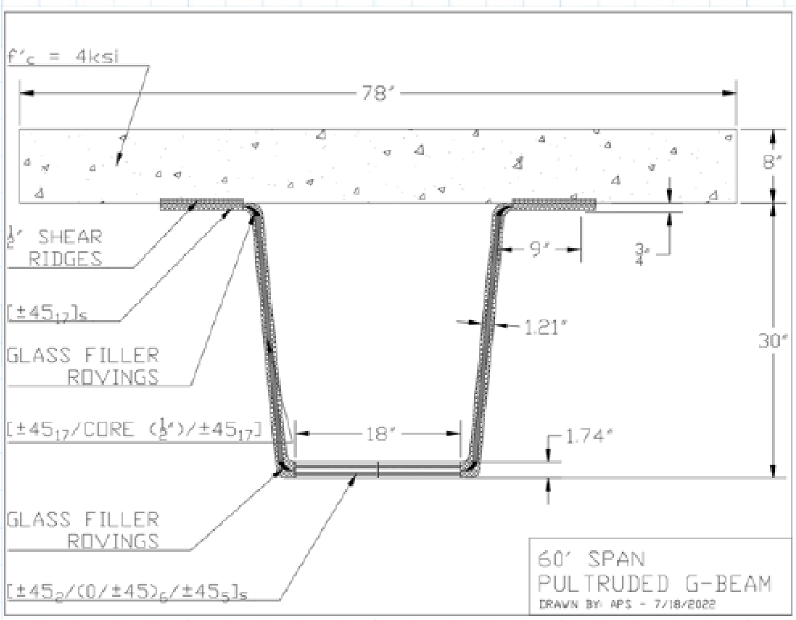
Resulting Section:

Depth: 30"

Bottom Flange:
18" Wide
1.74" Thick

Webs:
0.21" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



Minimum Girder Depth

40' Span

$\phi M_n := 2440\ ft \cdot kip$ $\quad$ $\text{if}\left(\phi M_n \geq M_{u40}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\phi V_n := 148.7\ kip$ $\quad$ $\text{if}\left(\phi V_n \geq V_{u40}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\Delta := 0.448\ in$ $\quad$ $\text{if}\left(\Delta \leq \Delta_{40}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.399\ in \qquad h_w := 15.51\ in \qquad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 13.656\ \frac{kip}{in}$$

Results from 24" and 36" nomographs

$r_1 := 0.25 \qquad h_1 := 24\ in$

$r_2 := 0.25 \qquad h_2 := 36\ in$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot \left(r_2 - r_1\right) = 0.25 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.266\ in$$

Say:
$t_{core} := 0.5\ in$

40' span uses a CIP deck. Prior to curing, girder is subject to top-flange compression failure and Flange Local Buckling

Top Flange Compression

$E_{tf} := 2069\ ksi \qquad n_{tf} := 0.1414 \qquad b_{tf} := 18\ in \qquad t_{tf} := 0.756\ in \quad S_x := 41.14\ in^3$

$I_{NC} := 649.04\ in^4 \quad d := 18\ in \qquad y_{NC} := 2.22\ in$

$$M_{PC} := \frac{E_{tf} \cdot \varepsilon_{1c} \cdot I_{NC}}{n_{tf} \cdot \left(d - y_{NC}\right)} = 601.832\ ft \cdot kip$$

$\phi M_{PC} := 0.75 \cdot M_{PC} = 451.374\ ft \cdot kip$ $\quad$ $\text{if}\left(\phi M_{PC} \geq \gamma M_{DC40}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

Flange Local Buckling

Assume:
- Girder acts as a doubly-symetric I-section such that AISC F.4 is Applicable
- Web is not slender
- Shear Ridges are not effective in resisting FLB
- Flanges are slender

$$k_c := \frac{4}{\sqrt{\frac{h_w}{2 \cdot t_{fs} + t_{core}}}} = 1.157 \qquad k_c := \text{if}\left(k_c > 0.76, 0.76, k_c\right) = 0.76$$

$$\lambda := \frac{b_{tf}}{2 \cdot t_{tf}} = 11.905$$

$$M_{FLB} := \frac{0.9 \cdot E_{tf} \cdot k_c \cdot S_x}{n_{tf} \cdot \lambda^2} = 242.108 \; (\boldsymbol{ft \cdot kip})$$

$$\phi M_{FLB} := 0.75 \cdot M_{FLB} = 181.581 \; \boldsymbol{ft \cdot kip} \qquad \text{if}\left(\phi M_{FLB} \geq \gamma M_{DC40}, \text{“OK”}, \text{“NG”}\right) = \text{“OK”}$$

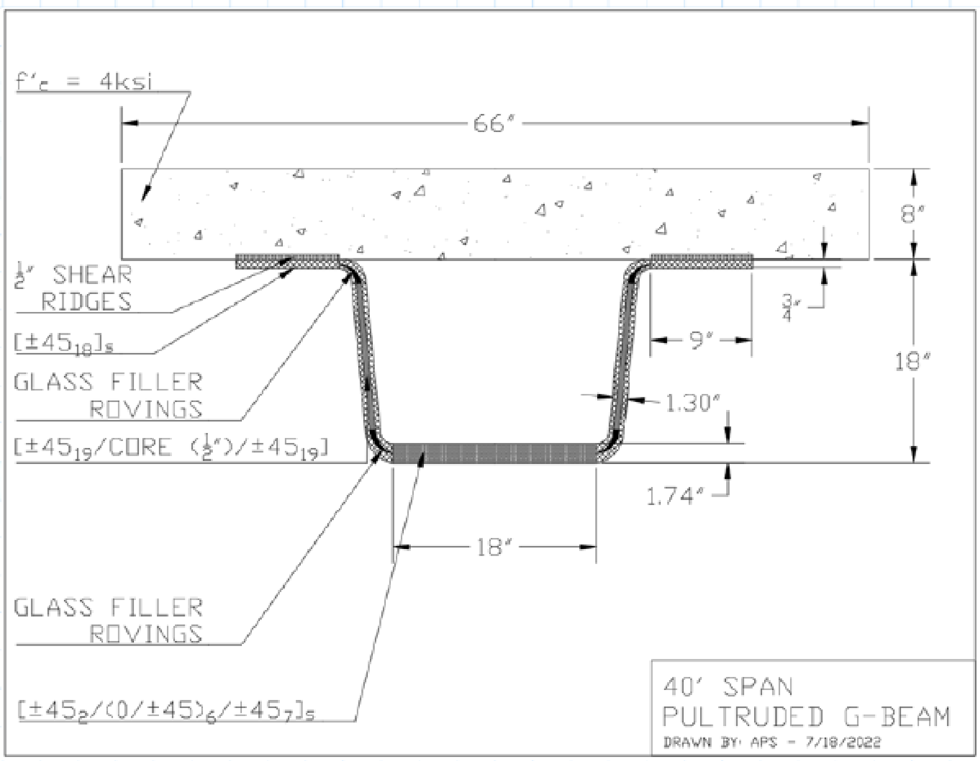
Resulting Section:

Depth: 18"

Bottom Flange:
18" Wide
1.74" Thick

Webs:
1.30" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



50' Span

$\phi M_n := 3510\ ft \cdot kip$ $\quad$ $\text{if}\left(\phi M_n \geq M_{u50}, \text{“OK”}, \text{“NG”}\right) = \text{“OK”}$

$\phi V_n := 159.5\ kip$ $\quad$ $\text{if}\left(\phi V_n \geq V_{u50}, \text{“OK”}, \text{“NG”}\right) = \text{“OK”}$

$\Delta := 0.588\ in$ $\quad$ $\text{if}\left(\Delta \leq \Delta_{50}, \text{“OK”}, \text{“NG”}\right) = \text{“OK”}$

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.378\ in \qquad h_w := 19.56\ in \qquad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 10.828\ \frac{kip}{in}$$

Results from 24" and 36" nomographs

$r_1 := .25 \qquad h_1 := 24\ in$

$r_2 := .25 \qquad h_2 := 36\ in$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot \left(r_2 - r_1\right) = 0.25 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.252\ in$$

Say:
$t_{core} := 0.5\ in$

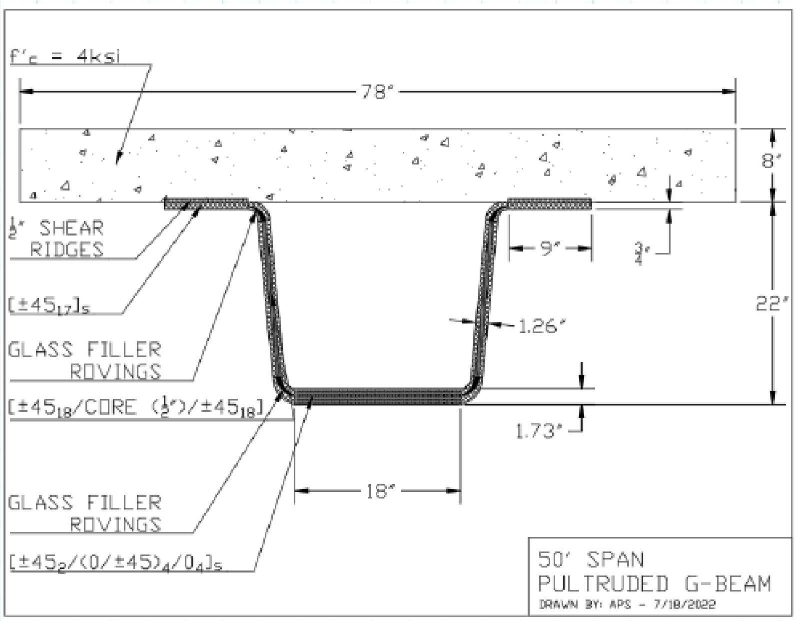
Resulting Section:

Depth 22"

Bottom Flange:
18" Wide
1.73" Thick

Webs:
1.26" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



60' Span

$\phi M_n := 4380\ ft \cdot kip$ $\quad$ $\text{if}\left(\phi M_n \ge M_{u60}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\phi V_n := 188\ kip$ $\quad$ $\text{if}\left(\phi V_n \ge V_{u60}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\Delta := 0.687\ in$ $\quad$ $\text{if}\left(\Delta \le \Delta_{60}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.378\ in \qquad h_w := 25.54\ in \qquad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 8.293\ \frac{kip}{in}$$

Results from 24" and 36" nomographs

$r_1 := .25 \qquad h_1 := 24\ in$

$r_2 := .4 \qquad h_2 := 36\ in$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot \left(r_2 - r_1\right) = 0.269 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.279\ in$$

Say:

$t_{core} := 0.5\ in$

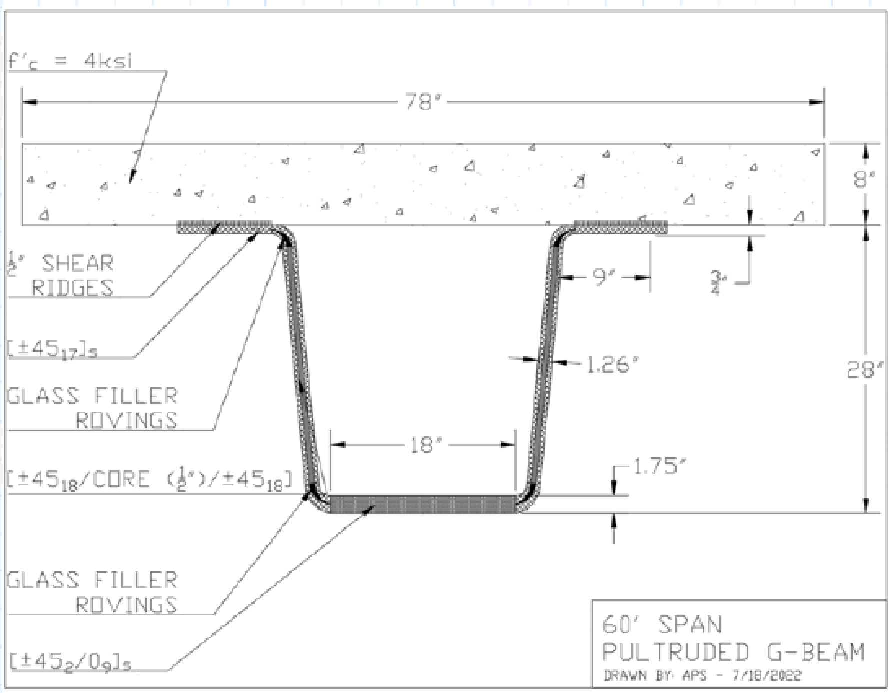
Resulting Section:

Depth: 28"

Bottom Flange:
18" Wide
1.75" Thick

Webs:
1.26" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



II Girder Design

Distribution Factors - Use AASHTO I-Beam DFs - Assume 3rd Term in Equation = 1.02 (Table 4.6.2.2.1-2)

40' Span

$S := 5.5\ ft \qquad L := 40\ ft$

Moment

$$g_1 := 0.06 + \left(\frac{S}{14\ ft}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot 1.02 = 0.447$$

$$g_2 := 0.075 + \left(\frac{S}{9.5\ ft}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot 1.02 = 0.527 \qquad g_{40} := \max\left(g_1, g_2\right) = 0.527$$

Shear

$$g_1 := 0.36 + \frac{S}{25\ ft} = 0.58$$

$$g_2 := 0.2 + \frac{S}{12\ ft} - \left(\frac{S}{35\ ft}\right)^2 = 0.634 \qquad g_{v40} := \max\left(g_1, g_2\right) = 0.634$$

50' Span

$S := 6.5\ ft \qquad L := 50\ ft$

Moment

$$g_1 := 0.06 + \left(\frac{S}{14\ ft}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot 1.02 = 0.467$$

$$g_2 := 0.075 + \left(\frac{S}{9.5\ ft}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot 1.02 = 0.55 \qquad g_{50} := \max\left(g_1, g_2\right) = 0.55$$

Shear

$$g_1 := 0.36 + \frac{S}{25\ ft} = 0.62$$

$$g_2 := 0.2 + \frac{S}{12\ ft} - \left(\frac{S}{35\ ft}\right)^2 = 0.707 \qquad g_{v50} := \max\left(g_1, g_2\right) = 0.707$$

60' Span

$S := 6.5\ \boldsymbol{ft}$ $\quad L := 60\ \boldsymbol{ft}$

Moment

$$g_1 := 0.06 + \left(\frac{S}{14\ \boldsymbol{ft}}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot 1.02 = 0.445$$

$$g_2 := 0.075 + \left(\frac{S}{9.5\ \boldsymbol{ft}}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot 1.02 = 0.525 \qquad g_{60} := \max\left(g_1, g_2\right) = 0.525$$

Shear

$$g_1 := 0.36 + \frac{S}{25\ \boldsymbol{ft}} = 0.62$$

$$g_2 := 0.2 + \frac{S}{12\ \boldsymbol{ft}} - \left(\frac{S}{35\ \boldsymbol{ft}}\right)^2 = 0.707 \qquad g_{v60} := \max\left(g_1, g_2\right) = 0.707$$

Loads

Girder Dead Load

$A_{frp} := 18\ in \cdot 1.75\ in + 4\ in \cdot 28\ in + 18\ in \cdot 1.25\ in = 1.153\ ft^2$

$w_{frp} := 0.1156\ \frac{kip}{ft^3} \cdot A_{frp} = 0.133\ \frac{kip}{ft}$

40' Span $L := 40\ ft$

Dead Loads

$w_{deck} := 0.15\ \frac{kip}{ft^3} \cdot 5.5\ ft \cdot 8\ in = 0.55\ \frac{kip}{ft}$

$w_{ws} := 0.145\ \frac{kip}{ft^3} \cdot 5.5\ ft \cdot 3\ in = 0.199\ \frac{kip}{ft}$

$w_{DC} := w_{deck} + w_{frp} = 0.683\ \frac{kip}{ft}$

$M_{DC40} := \frac{w_{DC} \cdot L^2}{8} = 136.652\ ft \cdot kip$ $\quad V_{DC40} := w_{DC} \cdot \frac{L}{2} = 13.665\ kip$

$M_{DW40} := \frac{w_{ws} \cdot L^2}{8} = 39.875\ ft \cdot kip$ $\quad V_{DW40} := w_{ws} \cdot \frac{L}{2} = 3.988\ kip$

Live Load

$M := \left(1.33 \cdot 32\ kip \cdot \left(\frac{L}{4} + \frac{0.5 \cdot L - 14\ ft}{L} \cdot \frac{L}{2} \cdot \left(1 + \frac{8}{32}\right)\right) + \frac{.64 \cdot kip}{ft} \cdot \frac{L^2}{8}\right) \cdot 1.25 = 891.5\ ft \cdot kip$

$V := \left(1.33 \cdot 32\ kip \cdot \left(1 + \frac{L - 14\ ft}{L} \cdot \left(1 + \frac{8}{32}\right)\right) + 0.64\ \frac{kip}{ft} \cdot \frac{L}{2}\right) \cdot 1.25 = 112.425\ kip$

$M_{LL40} := M \cdot g_{40} = 469.828\ ft \cdot kip$

$V_{LL40} := V \cdot g_{v40} = 71.237\ kip$

$M_{u40} := 1.75 \cdot M_{LL40} + 1.5 \cdot M_{DW40} = 882.012\ ft \cdot kip$

$\gamma M_{DC40} := 1.25 \cdot M_{DC40} = 170.815\ (ft \cdot kip)$

$V_{u40} := 1.75 \cdot V_{LL40} + 1.25 \cdot V_{DC40} + 1.5 \cdot V_{DW40} = 147.727\ kip$

$\Delta_{40} := \frac{L}{1000} = 0.48\ in$

50' Span $L := 50\ ft$

Dead Loads

$$w_{deck} := 0.15\ \frac{kip}{ft^3} \cdot 6.5\ ft \cdot 8\ in = 0.65\ \frac{kip}{ft}$$

$$w_{ws} := 0.145\ \frac{kip}{ft^3} \cdot 6.5\ ft \cdot 3\ in = 0.236\ \frac{kip}{ft}$$

$$w_{DC} := w_{deck} + w_{frp} = 0.783\ \frac{kip}{ft}$$

$$M_{DC50} := \frac{w_{DC} \cdot L^2}{8} = 244.769\ ft \cdot kip$$

$$M_{DW50} := \frac{w_{ws} \cdot L^2}{8} = 73.633\ ft \cdot kip$$

$$V_{DC50} := w_{DC} \cdot \frac{L}{2} = 19.582\ kip$$

$$V_{DW50} := w_{ws} \cdot \frac{L}{2} = 5.891\ kip$$

Live Loads

$$M := \left(1.33 \cdot 32\ kip \cdot \left(\frac{L}{4} + \frac{0.5 \cdot L - 14\ ft}{L} \cdot \frac{L}{2} \cdot \left(1 + \frac{8}{32}\right)\right) + \frac{.64 \cdot kip}{ft} \cdot \frac{L^2}{8}\right) \cdot 1.25 = \left(1.281 \cdot 10^3\right)\ ft \cdot kip$$

$$V := \left(1.33 \cdot 32\ kip \cdot \left(1 + \frac{L - 14\ ft}{L} \cdot \left(1 + \frac{8}{32}\right)\right) + 0.64\ \frac{kip}{ft} \cdot \frac{L}{2}\right) \cdot 1.25 = 121.08\ kip$$

$$M_{LL50} := M \cdot g_{50} = 704.644\ ft \cdot kip$$

$$V_{LL50} := V \cdot g_{v50} = 85.625\ kip$$

$$M_{u50} := 1.75 \cdot M_{LL50} + 1.5 \cdot M_{DW50} = \left(1.344 \cdot 10^3\right)\ ft \cdot kip$$

$$\gamma M_{DC50} := 1.25 \cdot M_{DC50} = 305.961\ (ft \cdot kip)$$

$$V_{u50} := 1.75 \cdot V_{LL40} + 1.25 \cdot V_{DC50} + 1.5 \cdot V_{DW50} = 157.977\ kip$$

$$\Delta_{50} := \frac{L}{1000} = 0.6\ in$$

60' Span $L := 60\ ft$

Dead Loads

$$w_{deck} := 0.15\ \frac{kip}{ft^3} \cdot 6.5\ ft \cdot 8\ in = 0.65\ \frac{kip}{ft}$$

$$w_{ws} := 0.145\ \frac{kip}{ft^3} \cdot 6.5\ ft \cdot 3\ in = 0.236\ \frac{kip}{ft}$$

$$w_{DC} := w_{deck} + w_{frp} = 0.783\ \frac{kip}{ft}$$

$$M_{DC60} := \frac{w_{DC} \cdot L^2}{8} = 352.468\ ft \cdot kip$$

$$M_{DW60} := \frac{w_{ws} \cdot L^2}{8} = 106.031\ ft \cdot kip$$

$$V_{DC60} := w_{DC} \cdot \frac{L}{2} = 23.498\ kip$$

$$V_{DW60} := w_{ws} \cdot \frac{L}{2} = 7.069\ kip$$

Live Loads

$$M := \left(1.33 \cdot 32\ kip \cdot \left(\frac{L}{4} + \frac{0.5 \cdot L - 14\ ft}{L} \cdot \frac{L}{2} \cdot \left(1 + \frac{8}{32}\right)\right) + \frac{.64 \cdot kip}{ft} \cdot \frac{L^2}{8}\right) \cdot 1.25 = \left(1.69 \cdot 10^3\right)\ ft \cdot kip$$

$$V := \left(1.33 \cdot 32\ kip \cdot \left(1 + \frac{L - 14\ ft}{L} \cdot \left(1 + \frac{8}{32}\right)\right) + 0.64\ \frac{kip}{ft} \cdot \frac{L}{2}\right) \cdot 1.25 = 128.183\ kip$$

$$M_{LL60} := M \cdot g_{60} = 887.061\ ft \cdot kip$$

$$V_{LL60} := V \cdot g_{v60} = 90.648\ kip$$

$$M_{u60} := 1.75 \cdot M_{LL60} + 1.5 \cdot M_{DW60} = \left(1.711 \cdot 10^3\right)\ ft \cdot kip$$

$$\gamma M_{DC60} := 1.25 \cdot M_{DC60} = 440.584\ (ft \cdot kip)$$

$$V_{u60} := 1.75 \cdot V_{LL60} + 1.25 \cdot V_{DC60} + 1.5 \cdot V_{DW60} = 198.61\ kip$$

$$\Delta_{60} := \frac{L}{1000} = 0.72\ in$$

Design Checks

Minimum Bottom Flange Thickness
40' Span

$\phi M_n := 2600\ ft \cdot kip$ $\qquad$ if $(\phi M_n \geq M_{u40}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

$\phi V_n := 153\ kip$ $\qquad$ if $(\phi V_n \geq V_{u40}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

$\Delta := 0.47\ in$ $\qquad$ if $(\Delta \leq \Delta_{40}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.357\ in \qquad h_w := 17.7\ in \qquad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 11.923\ \frac{kip}{in}$$

Results from 24" and 36" nomographs

$r_1 := 0.33 \qquad h_1 := 24\ in$

$r_2 := 0.33 \qquad h_2 := 36\ in$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot (r_2 - r_1) = 0.33 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.352\ in$$

Say:
$t_{core} := 0.5\ in$

40' span uses a CIP deck. Prior to curing, girder is subject to top-flange compression failure and Flange Local Buckling

Top Flange Compression

$E_{tf} := 2069\ ksi \qquad n_{tf} := 0.1832 \qquad b_{tf} := 18\ in \qquad t_{tf} := 0.714\ in \quad S_x := 57.24\ in^3$

$I_{NC} := 984.14\ in^4 \quad d := 20\ in \qquad y_{NC} := 2.806\ in$

$$M_{PC} := \frac{E_{tf} \cdot \varepsilon_{1c} \cdot I_{NC}}{n_{tf} \cdot (d - y_{NC})} = 646.42\ ft \cdot kip$$

$\phi M_{PC} := 0.75 \cdot M_{PC} = 484.815\ ft \cdot kip$ $\qquad$ if $(\phi M_{PC} \geq \gamma M_{DC40}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

Flange Local Buckling

Assume:
- Girder acts as a doubly-symetric I-section such that AISC F.4 is Applicable
- Web is not slender
- Shear Ridges are not effective in resisting FLB
- Flanges are slender

$$k_c := \frac{4}{\sqrt{\frac{h_w}{2 \cdot t_{fs} + t_{core}}}} = 1.048 \qquad k_c := \text{if}\left(k_c > 0.76, 0.76, k_c\right) = 0.76$$

$$\lambda := \frac{b_{tf}}{2 \cdot t_{tf}} = 12.605$$

$$M_{FLB} := \frac{0.9 \cdot E_{tf} \cdot k_c \cdot S_x}{n_{tf} \cdot \lambda^2} = 231.911 \ (\boldsymbol{ft} \cdot \boldsymbol{kip})$$

$$\phi M_{FLB} := 0.75 \cdot M_{FLB} = 173.933 \ \boldsymbol{ft} \cdot \boldsymbol{kip} \qquad \text{if}\left(\phi M_{FLB} \geq \gamma M_{DC40}, \text{“OK”}, \text{“NG”}\right) = \text{“OK”}$$

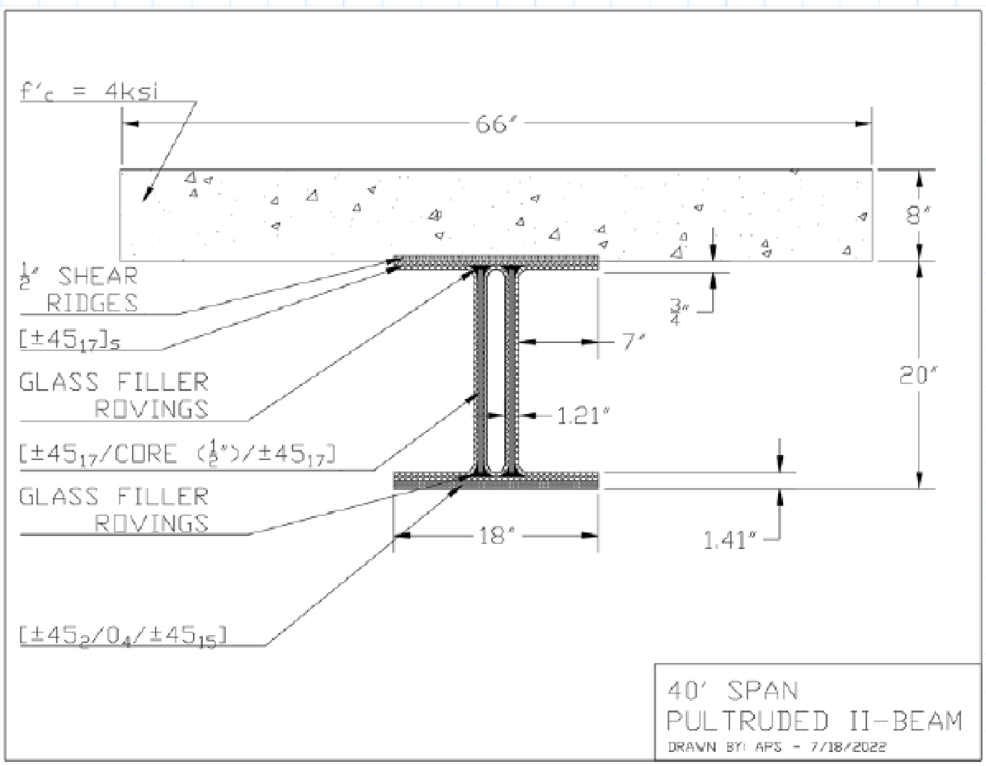
Resulting Section:

Depth: 20"

Bottom Flange:
18" Wide
1.41" Thick

Webs:
1.21" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



50' Span

$\phi M_n := 4010\ ft \cdot kip$ $\quad$ $\text{if}\left(\phi M_n \geq M_{u50}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\phi V_n := 185\ kip$ $\quad$ $\text{if}\left(\phi V_n \geq V_{u50}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\Delta := 0.572\ in$ $\quad$ $\text{if}\left(\Delta \leq \Delta_{50}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.357\ in \qquad h_w := 23.65\ in \qquad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 8.923\ \frac{kip}{in}$$

Results from 24" and 36" nomographs

$r_1 := 0.25 \qquad h_1 := 24\ in$

$r_2 := 0.25 \qquad h_2 := 36\ in$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot \left(r_2 - r_1\right) = 0.25 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.238\ in$$

Say: (For consistency)
$t_{core} := 0.5\ in$

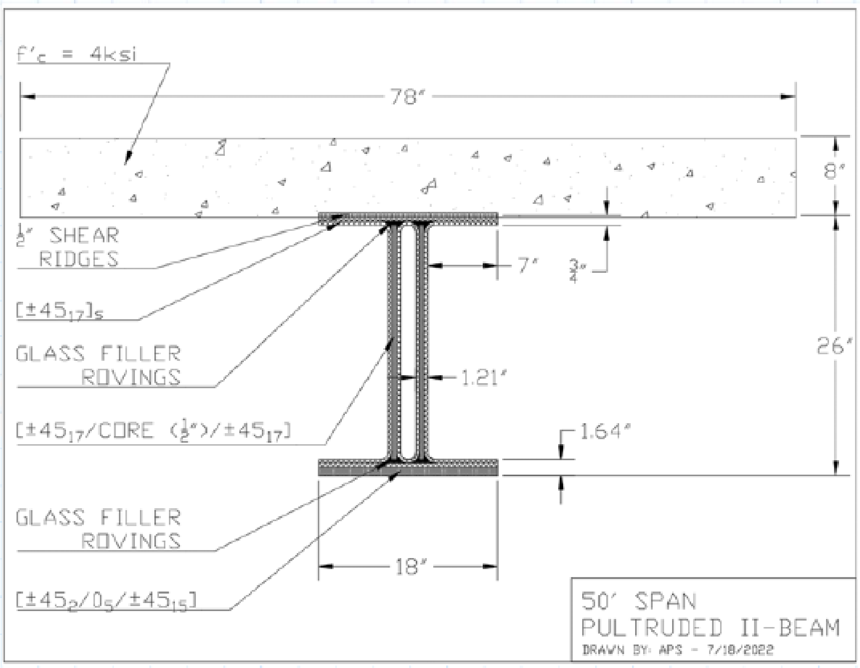
Resulting Section:

Depth 26"

Bottom Flange:
18" Wide
1.64" Thick

Webs:
1.21" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



60' Span

$\phi M_n := 4640\ \boldsymbol{ft \cdot kip}$ $\quad$ $\text{if}\left(\phi M_n \ge M_{u60}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\phi V_n := 200\ \boldsymbol{kip}$ $\quad$ $\text{if}\left(\phi V_n \ge V_{u60}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\Delta := 0.7512\ \boldsymbol{in}$ $\quad$ $\text{if}\left(\Delta \le \Delta_{60}, \text{"OK"}, \text{"NG"}\right) = \text{"NG"}$

Section does not meet deflection criteria, but only by a small amount. Call it OK.

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.357\ \boldsymbol{in} \qquad h_w := 27.54\ \boldsymbol{in} \qquad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 7.663\ \frac{\boldsymbol{kip}}{\boldsymbol{in}}$$

Results from 24" and 36" nomographs

$r_1 := 0.3 \qquad h_1 := 24\ \boldsymbol{in}$

$r_2 := 0.48 \qquad h_2 := 36\ \boldsymbol{in}$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot \left(r_2 - r_1\right) = 0.353 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.39\ \boldsymbol{in}$$

Say:
$t_{core} := 0.5\ \boldsymbol{in}$

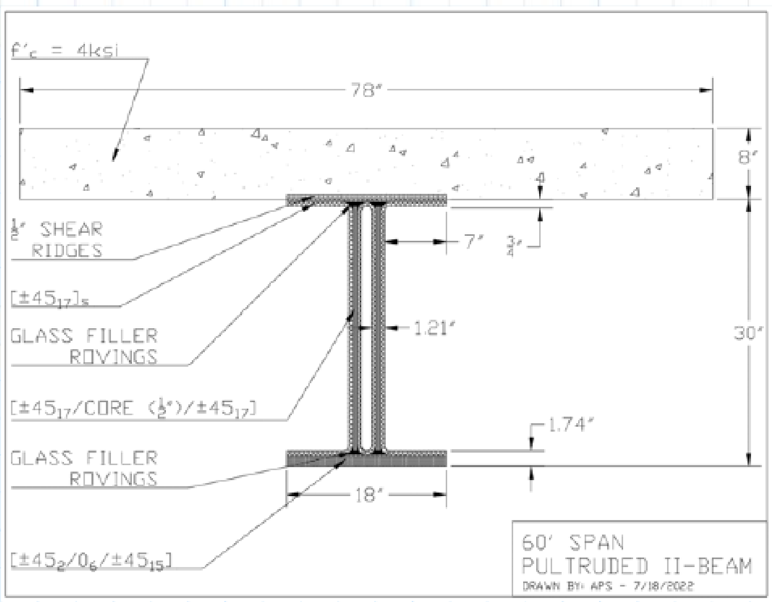
Resulting Section:

Depth: 30"

Bottom Flange:
18" Wide
1.74" Thick

Webs:
1.21" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



Minimum Girder Depth

40' Span

$\phi M_n := 2440\ \mathbf{ft} \cdot \mathbf{kip}$ $\qquad$ $\text{if}\left(\phi M_n \ge M_{u40}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\phi V_n := 148.7\ \mathbf{kip}$ $\qquad$ $\text{if}\left(\phi V_n \ge V_{u40}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\Delta := 0.448\ \mathbf{in}$ $\qquad$ $\text{if}\left(\Delta \le \Delta_{40}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.399\ \mathbf{in} \qquad h_w := 15.51\ \mathbf{in} \qquad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 13.607\ \frac{\mathbf{kip}}{\mathbf{in}}$$

Results from 24" and 36" nomographs

$r_1 := 0.25 \qquad h_1 := 24\ \mathbf{in}$

$r_2 := 0.25 \qquad h_2 := 36\ \mathbf{in}$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot \left(r_2 - r_1\right) = 0.25 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.266\ \mathbf{in}$$

Say:
$t_{core} := 0.5\ \mathbf{in}$

40' span uses a CIP deck. Prior to curing, girder is subject to top-flange compression failure and Flange Local Buckling

Top Flange Compression

$E_{tf} := 2069\ \mathbf{ksi} \qquad n_{tf} := 0.1414 \qquad b_{tf} := 18\ \mathbf{in} \qquad t_{tf} := 0.756\ \mathbf{in} \quad S_x := 41.14\ \mathbf{in}^3$

$I_{NC} := 649.04\ \mathbf{in}^4 \quad d := 18\ \mathbf{in} \qquad y_{NC} := 2.22\ \mathbf{in}$

$$M_{PC} := \frac{E_{tf} \cdot \varepsilon_{1c} \cdot I_{NC}}{n_{tf} \cdot \left(d - y_{NC}\right)} = 601.832\ \mathbf{ft} \cdot \mathbf{kip}$$

$\phi M_{PC} := 0.75 \cdot M_{PC} = 451.374\ \mathbf{ft} \cdot \mathbf{kip}$ $\qquad$ $\text{if}\left(\phi M_{PC} \ge \gamma M_{DC40}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

Flange Local Buckling

Assume:
- Girder acts as a doubly-symetric I-section such that AISC F.4 is Applicable
- Web is not slender
- Shear Ridges are not effective in resisting FLB
- Flanges are slender

$$k_c := \frac{4}{\sqrt{\frac{h_w}{2 \cdot t_{fs} + t_{core}}}} = 1.157 \qquad k_c := \text{if}\left(k_c > 0.76, 0.76, k_c\right) = 0.76$$

$$\lambda := \frac{b_{tf}}{2 \cdot t_{tf}} = 11.905$$

$$M_{FLB} := \frac{0.9 \cdot E_{tf} \cdot k_c \cdot S_x}{n_{tf} \cdot \lambda^2} = 242.108 \; (ft \cdot kip)$$

$$\phi M_{FLB} := 0.75 \cdot M_{FLB} = 181.581 \; ft \cdot kip \qquad \text{if}\left(\phi M_{FLB} \ge \gamma M_{DC40}, \text{“OK”}, \text{“NG”}\right) = \text{“OK”}$$

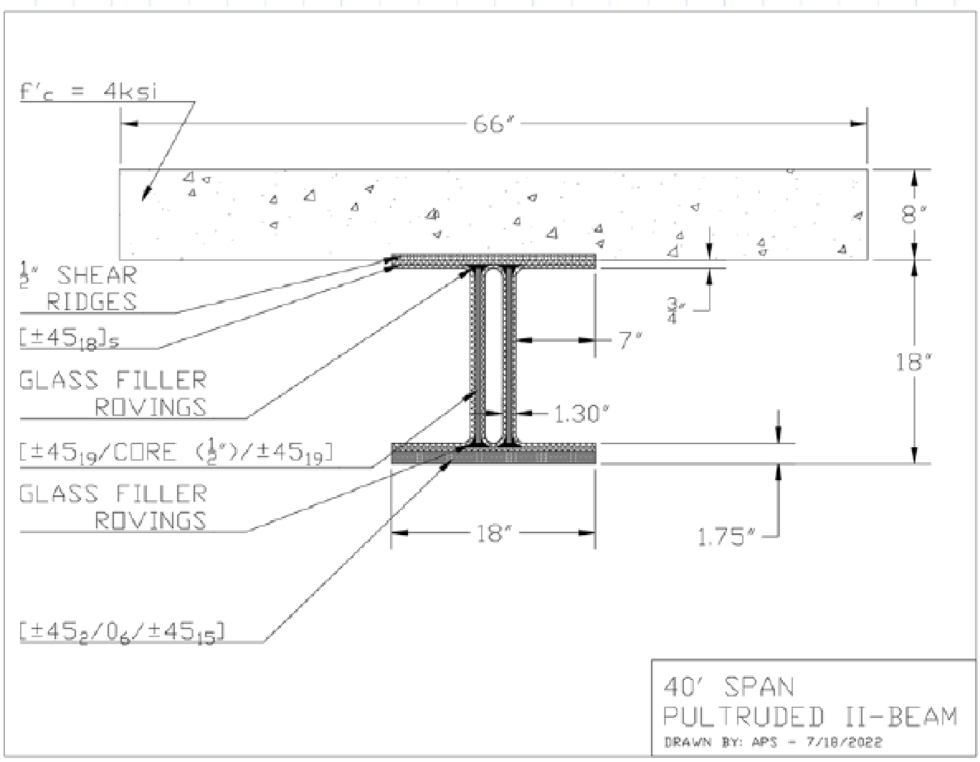
Resulting Section:

Depth: 18"

Bottom Flange:
18" Wide
1.74" Thick

Webs:
1.30" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



50' Span

$\phi M_n := 3510\ ft \cdot kip$ $\qquad$ $\text{if}\left(\phi M_n \ge M_{u50}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\phi V_n := 159.5\ kip$ $\qquad$ $\text{if}\left(\phi V_n \ge V_{u50}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

$\Delta := 0.588\ in$ $\qquad$ $\text{if}\left(\Delta \le \Delta_{50}, \text{"OK"}, \text{"NG"}\right) = \text{"OK"}$

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.378\ in \qquad h_w := 19.56\ in \qquad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 10.789\ \frac{kip}{in}$$

Results from 24" and 36" nomographs

$r_1 := .25 \qquad h_1 := 24\ in$

$r_2 := .25 \qquad h_2 := 36\ in$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot \left(r_2 - r_1\right) = 0.25 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.252\ in$$

Say:
$t_{core} := 0.5\ in$

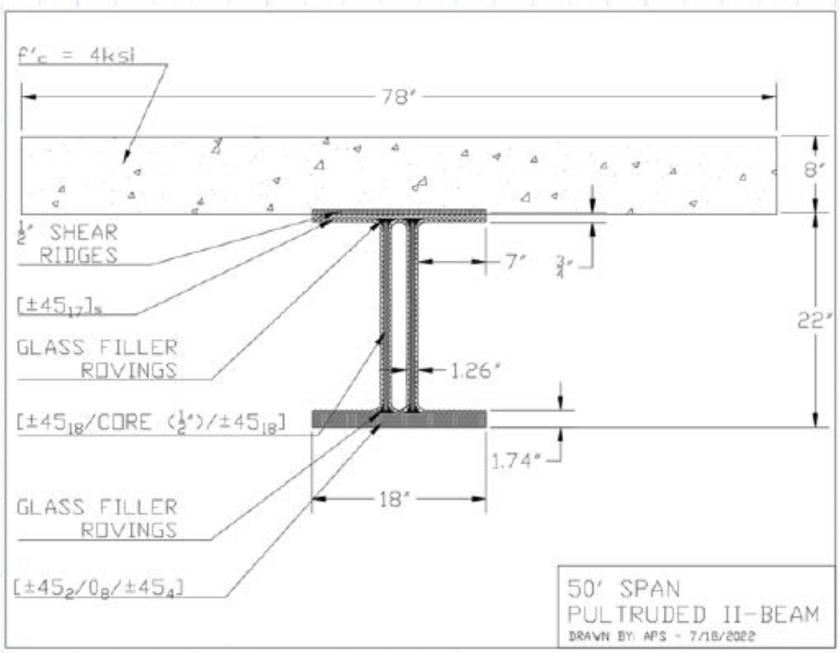
Resulting Section:

Depth 22"

Bottom Flange:
18" Wide
1.73" Thick

Webs:
1.26" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



60' Span

$\phi M_n := 4380\ ft \cdot kip$ $\quad$ if $(\phi M_n \geq M_{u60}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

$\phi V_n := 209\ kip$ $\quad$ if $(\phi V_n \geq V_{u60}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

$\Delta := 0.687\ in$ $\quad$ if $(\Delta \leq \Delta_{60}, \text{"OK"}, \text{"NG"}) = \text{"OK"}$

Shear Buckling - Choose Core Thickness

$t_{fs} := 0.378\ in \qquad h_w := 25.54\ in \qquad \phi_{VB} := 0.35$

$$N_{xy} := \frac{V_{u40}}{2 \cdot \phi_{VB} \cdot h_w} = 8.263\ \frac{kip}{in}$$

Results from 24" and 36" nomographs

$r_1 := .25 \qquad h_1 := 24\ in$

$r_2 := .4 \qquad h_2 := 36\ in$

$$r := r_1 + \frac{h_w - h_1}{h_2 - h_1} \cdot (r_2 - r_1) = 0.269 \qquad t_{core} := \frac{2 \cdot t_{fs} \cdot r}{1 - r} = 0.279\ in$$

Say:
$t_{core} := 0.5\ in$

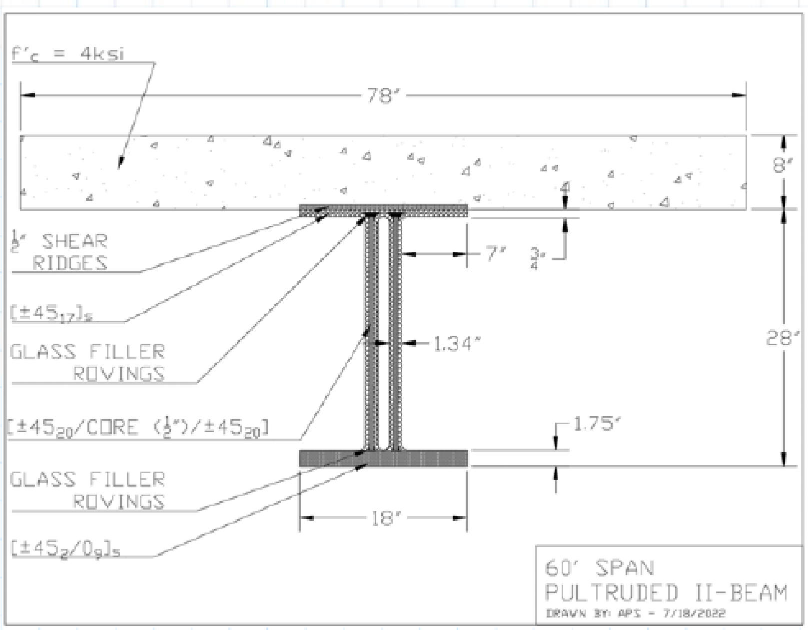
Resulting Section:

Depth: 28"

Bottom Flange:
18" Wide
1.75" Thick

Webs:
1.26" Thick
0.5" Core

Top Flange:
18" Wide
3/4" Thick
1/2" Ridges



Appendix B: Flexural Strength and Stiffness MATLAB Code

B1: Driver Function

```
function [phiMnFull,modef,phiVn,vmax,vlim] = CT_design_small

% Factored dead-load moment
MDC_Factored = 170.75*12;
% Reduced
f6 = .65*10;
% Mark interior Girder Sectin
gird = 2;
% Establish a global structure to hold input values
global inp EI L wDC wDW g girds
inp = struct;
% Number of girders
girds = 6;

%% Inputs and Initial Calculations
% Height of FRP section (in)
dfrp = 18;
% Thickness of Slab (in)
tc = 8;
% Total section height
D = dfrp+tc;
% Web, Bottom Flange, and Top Flange Bending Modulus (ksi) and Thickness
% (in)
[Ew,tw] = web_E_s;
[Ebf,tbf] = bf_E_s;
[Etf,ttf] = tf_E_s;
% Rupture stress of bottom flange main reinforcement (ksi)
F1t = 379*.85;
% Tensile modulus of bottom flange main reinforcement (ksi)
E1t = 22.77*1000;
% Bottom Flange Width (in)
bbf = 18;
% Top Flange Width (in)
btf = 18;
% Deck Width (in)
bc = 66;
% Sidewalk Width (in)
bsw = 0;
% Curb Width (in)
bcurb = 0;
% Curb Depth (in)
tcurb = 0;
hw = dfrp-tbf-ttf;
% Modular Rations
Eref = Ebf;
nbf = Ebf/Eref;
nw = Ew/Eref;
ntf = Etf/Eref;
% Transformed Section Analyis
```

```
Abf = nbf*bbf*tbf;
ybf = tbf/2;
Ibf = nbf/12*bbf*tbf^3;
Aweb = nw*tw*hw;
yweb = tbf+hw/2;
Iweb = nw*tw/12*hw^3;
Atf = ntf*btf*ttf;
ytf = dfrp-ttf/2;
Itf = ntf/12*btf*ttf^3;
Afrp = Abf+Aweb+Atf;
ybar = (Abf*ybf+Aweb*yweb+Atf*ytf)/Afrp;
I0 = Ibf+Iweb+Itf;
PA = Abf*(ybf-ybar)^2+Aweb*(yweb-ybar)^2+Atf*(ytf-ybar)^2;
Ifrp = I0+PA;
SNC = Ifrp/ybar;
Stop = Ifrp/(dfrp-ybar);

% Strain from dead-load moment
eps_DC = MDC_Factored/SNC/Eref;
% Carbon rupture strain less dead-laod strain
epsrupt = F1t/E1t-eps_DC;

% Material Parameters
% Concrete Compressive Strength (ksi)
fpc = 4;
% Steel Elastic Mod. (ksi)
Es = 29000;
% Steel Yield Strength (ksi)
fy = 60;
% Conc. Elastic Mod. (ksi)
Ec = 1820*sqrt(fpc);
% Conc modular ratio
nc = Ec/Eref;

% Steel
% Area of reinforcement (in^2)
Ab = 0.3068;
nums = 0;
As = Ab*nums;
% Height of Steel Centroid (in)
ds = D-tc/2;

% Package inputs for Optimization
inp.tbf = tbf; inp.Ebf = Ebf; inp.bbf = bbf;inp.ttf = ttf; inp.tw = tw;
inp.Ew = Ew; inp.Etf = Etf;inp.btf = btf; inp.fpc = fpc; inp.Ec = Ec;
inp.bc = bc; inp.D = D;inp.tc = tc; inp.ds = ds; inp.Es = Es; inp.As = As;
inp.fy = fy; inp.gird = gird; inp.bsw = bsw; inp.bcurb = bcurb;
inp.tcurb = tcurb;inp.dfrp = dfrp;

%% Moment-Curvature Analysis %%

% Range of Curvatures (1/in)
k = 0:1e-7:.01;
% Function handle to function that sums internal forces for equilibrium
fun = @get_ybar;
```

```
% Allocate memory
M = zeros(size(k));
boteps = M;
topeps = M;
ybar = M;
% Create a stop flag
stopper = 0;
% Create an iteration counter
ii = 0;
% While the section has not failed
while stopper == 0
%     Count the iteration
    ii = ii+1;
%     Get the current value of curvature
    inp.k = k(ii);
%     Find the neutral axis height leading to equilibrium by nonlinear
%     optimization
    ybar(ii) = fminsearch(fun,D);
%     Discretize the section
    n = 1000;
    tn = D/n;
    y = 0:tn:D;
%     For each layer of the section
    for jj = 1:length(y)
%         Calculate strain
        eps = -k(ii)*(y(jj)-ybar(ii));
%         Determine if the section has failed
        if jj == 1
            boteps(ii) = eps;
            if eps >= epsrupt
                stopper = 1;
                modef = 'Bot Flange Rupture';
            end
        elseif jj == length(y)
            topeps(ii) = eps;
            if eps < -0.003
                stopper = 1;
                modef = 'Concrete Crushing';
            end
        end
%         Calculate moment acting on the section
        if y(jj) <= tbf
            f = Ebf*eps;
            A = bbf*tn;
        elseif y(jj) <= dfrp-ttf
            f = Ew*eps;
            A = tw*tn;
        elseif y(jj) <= dfrp
            f = Etf*eps;
            A = btf*tn;
        elseif y(jj) <= dfrp+0.5
            f = 0;
            A = 0;
        elseif y(jj) <= dfrp+0.5+tc
            f = Hognestad(eps,fpc,Ec);
```

```
            A = bc*tn;
        end
        F = f*A;
        M(ii) = M(ii)+F*(D-y(jj));
    end

    epss = -k(ii)*(ds-ybar(ii));
    fs = Es*epss;
    if abs(fs) > fy
        fs = fy*sign(fs);
    end
    Fs = fs*As;
    M(ii) = M(ii)+Fs*(ds);
end
Msave = M(1:ii);
Mnfull = Msave(end);

% Find factored moment resistance of the section
if strcmp(modef,'Bot Flange Rupture')
    phiMnFull = .75*Mnfull/12;
elseif strcmp(modef,'Concrete Crushing')
    phiMnFull = .75*Mnfull/12;
end

%% Shear Resistance %%

% Perform transformed section analysis adding the cured concrete deck
Ac = nc*bc*tc;
yc = D-tc/2;
Ic = nc/12*bc*tc^3;
Ay = Abf*ybf+Aweb*yweb+Atf*ytf+Ac*yc;
A = Abf+Aweb+Atf+Ac;
ybar = Ay/A;
I0 = Ibf+Iweb+Itf+Ic;
PA = Abf*(ybf-ybar)^2+Aweb*(yweb-ybar)^2+Atf*(ytf-ybar)^2+Ac*(yc-ybar);
Icomp = I0+PA;

% Compute the first moment of area of the bottom flange and web about the
% neutral axis
if ybar < dfrp-ttf
    A = Abf+(ybar-tbf)*nweb*tw;
    yq = (Abf*ybf+(ybar-tbf)*nweb*tw*(ybar-tbf)/2)/A;
    Q = A*(ybar-yq);
else
    A = Abf+Aweb;
    yq = (Abf*ybf+Aweb*yweb)/A;
    Q = A*(ybar-yq);
end
% Compute the factored shear resistance
Vn = f6*Icomp*2*tw/Q;
phiVn = 0.75*Vn;

%% Deflection Calculation %%

% Calculate flexural rigidity
```

```
EI = Icomp*Eref;

% Span Length (ft)
L = 40*12;
% Dead laods
wDC = 0.683;
wDW = 0.2;
% Distribution Factor
g = 0.636;
% Get maximum deflection
[~,y] = getdisp;
vmax = max(y);
% Calculate Deflection limit
vlim = L/1000;
```

B2: web_E_s

```
function [Ex,t] = web_E_s

% EBX-2400 - Uniaxial Equivalent
% Modular Values (ksi)
E1b = 6.63*1000;
E2b = 1.04*1000;
nub = 0.25;
Gb = .69*1000;

db = 1-nub^2*E2b/E1b;
Qb = [E1b/db nub*E2b/db 0;
    nub*E2b/db  E2b/db 0;
    0 0 Gb];

% Core Material
% Modular Values (ksi)
E1c = 15.950;
E2c = 15.950;
nuc = 0.3;
Gc = 3.190;

dc = 1-nuc^2*E2c/E1c;
Qc = [E1c/dc nuc*E2c/dc 0;
    nuc*E2c/dc  E2c/dc 0;
    0 0 Gc];


% Number of Layers
layer = 19;

t = [0.0105*ones(layer*2,1);];
theta = [45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;
    -45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;
    45;-45;];
t = [t;flipud(t)];
theta = [theta flipud(theta)];
z = [0;cumsum(t)];
    thick = sum(t);
    z = z-thick/2;
% Create direction cosines of each lamina
m = cosd(theta);
n = sind(theta);
% Create Reuter Matrix and its Inverse
R = [1 0 0;
    0 1 0;
    0 0 2];
Rinv = inv(R);
% Allocate space for global stiffness matrix and laminar stiffness matrices
% in global coordinates
A = zeros(3);
B = A;
D = A;
```

```
% Qbar = zeros(3,3,length(t));
% For each lamina
for ii = 1:length(t)
%     Create the transformation matrix
    T = [m(ii)^2 n(ii)^2 2*m(ii)*n(ii);
        n(ii)^2 m(ii)^2 -2*m(ii)*n(ii);
        -m(ii)*n(ii) m(ii)*n(ii) m(ii)^2-n(ii)^2];
    Tinv = inv(T);

    if t(ii) == .75
        Q = Qc;
    elseif t(ii) == 0.15
        Q = Qt;
    else
        Q = Qb;
    end

%     Transform laminar stiffness to global coordinates
    Qbar = Tinv*Q*R*T*Rinv;
    A(1,1) = A(1,1)+Qbar(1,1)*(z(ii+1)-z(ii));
    A(1,2) = A(1,2)+Qbar(1,2)*(z(ii+1)-z(ii));
    A(1,3) = A(1,3)+Qbar(1,3)*(z(ii+1)-z(ii));
    A(2,1) = A(2,1)+Qbar(2,1)*(z(ii+1)-z(ii));
    A(2,2) = A(2,2)+Qbar(2,2)*(z(ii+1)-z(ii));
    A(2,3) = A(2,3)+Qbar(2,3)*(z(ii+1)-z(ii));
    A(3,1) = A(3,1)+Qbar(3,1)*(z(ii+1)-z(ii));
    A(3,2) = A(3,2)+Qbar(3,2)*(z(ii+1)-z(ii));
    A(3,3) = A(3,3)+Qbar(3,3)*(z(ii+1)-z(ii));

end
%  Collect global, stiffness matrix and global compliance matrix
t = sum(t);
% Equivalent Modulus (ksi)
Ex = (A(1,1)*A(2,2)-A(1,2)^2)/t/A(2,2);
```

B3: bf_E_s

```
function [Ex,t] = bf_E_s

% EBX-2400 - Uniaxial Equivalent
% Modular Values (ksi)
E1b = 6.63*1000;
E2b = 1.04*1000;
nub = 0.25;
Gb = .69*1000;

db = 1-nub^2*E2b/E1b;
Qb = [E1b/db nub*E2b/db 0;
    nub*E2b/db  E2b/db 0;
    0 0 Gb];

% Carbon
% Modular Values (ksi)
E1c = 21.74*1000;
E2c = 1.09*1000;
nuc = 0.238;
Gc = .69*1000;

dc = 1-nuc^2*E2c/E1c;
Qc = [E1c/dc nuc*E2c/dc 0;
    nuc*E2c/dc  E2c/dc 0;
    0 0 Gc];

% Layer Thickness (in)
t = [0.0105*ones(2*2,1);0.0923*ones(6,1);0.0105*ones(13*2,1);];
% Layer orientation (deg)
theta = [45;-45;45;-45;0;0;0;0;0;0;
    45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;
    45;-45;45;-45;45;-45;];
t = [t;flipud(t)];
theta = [theta flipud(theta)];

z = [0;cumsum(t)];
    thick = sum(t);
    z = z-thick/2;

% Create direction cosines of each lamina
m = cosd(theta);
n = sind(theta);
% Create Reuter Matrix and its Inverse
R = [1 0 0;
    0 1 0;
    0 0 2];
Rinv = inv(R);
% Allocate space for global stiffness matrix and laminar stiffness matrices
% in global coordinates
A = zeros(3);
Qbar = zeros(3,3,length(t));
% For each lamina
```

```
for ii = 1:length(t)
%     Create the transformation matrix
    T = [m(ii)^2 n(ii)^2 2*m(ii)*n(ii);
        n(ii)^2 m(ii)^2 -2*m(ii)*n(ii);
        -m(ii)*n(ii) m(ii)*n(ii) m(ii)^2-n(ii)^2];
    Tinv = inv(T);

    if t(ii) == .0923 || t(ii) == 0.0923/2
        Q = Qc;
    else
        Q = Qb;
    end

%     Transform laminar stiffness to global coordinates
    Qbar(:,:,ii) = Tinv*Q*R*T*Rinv;
%     Add lamina to global stiffness matrix
    A(1,1) = A(1,1)+Qbar(1,1,ii)*(z(ii+1)-z(ii));
    A(1,2) = A(1,2)+Qbar(1,2,ii)*(z(ii+1)-z(ii));
    A(1,3) = A(1,3)+Qbar(1,3,ii)*(z(ii+1)-z(ii));
    A(2,1) = A(2,1)+Qbar(2,1,ii)*(z(ii+1)-z(ii));
    A(2,2) = A(2,2)+Qbar(2,2,ii)*(z(ii+1)-z(ii));
    A(2,3) = A(2,3)+Qbar(2,3,ii)*(z(ii+1)-z(ii));
    A(3,1) = A(3,1)+Qbar(3,1,ii)*(z(ii+1)-z(ii));
    A(3,2) = A(3,2)+Qbar(3,2,ii)*(z(ii+1)-z(ii));
    A(3,3) = A(3,3)+Qbar(3,3,ii)*(z(ii+1)-z(ii));
end
%  Collect global, stiffness matrix and global compliance matrix
t = sum(t);
% Equivalent Modulus (ksi)
Ex = (A(1,1)*A(2,2)-A(1,2)^2)/t/A(2,2);
```

B4: tf_E_s

```
function [Ex,t] = tf_E_s
% EBX-2400 - Uniaxial Equivalent
% Modular Values (ksi)
E1b = 6.63*1000;
E2b = 1.04*1000;
nub = 0.25;
Gb = .69*1000;


db = 1-nub^2*E2b/E1b;
Qb = [E1b/db nub*E2b/db 0;
    nub*E2b/db  E2b/db 0;
    0 0 Gb];


% Number of Layers
layer = 12;
% layer = 8;
% Layer Thickness (in)
t = [.0105*ones(18*2,1)];
% Layer orientation (deg)
theta = [45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;
    45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;45;-45;];
t = [t;flipud(t)];
theta = [theta;flipud(theta)];

z = [0;cumsum(t)];
    thick = sum(t);
    z = z-thick/2;

% Create direction cosines of each lamina
m = cosd(theta);
n = sind(theta);
% Create Reuter Matrix and its Inverse
R = [1 0 0;
    0 1 0;
    0 0 2];
Rinv = inv(R);
% Allocate space for global stiffness matrix and laminar stiffness matrices
% in global coordinates
A = zeros(3);
Qbar = zeros(3,3,length(t));
% For each lamina
for ii = 1:length(t)
%     Create the transformation matrix
    T = [m(ii)^2 n(ii)^2 2*m(ii)*n(ii);
        n(ii)^2 m(ii)^2 -2*m(ii)*n(ii);
        -m(ii)*n(ii) m(ii)*n(ii) m(ii)^2-n(ii)^2];
    Tinv = inv(T);
    if t(ii) == .75
        Q = Qc;
    elseif t(ii) == 0.15
```

```
        Q = Qt;
    else
        Q = Qb;
    end
%     Transform laminar stiffness to global coordinates
    Qbar(:,:,ii) = Tinv*Q*R*T*Rinv;
%     Add lamina to global stiffness matrix
A(1,1) = A(1,1)+Qbar(1,1)*(z(ii+1)-z(ii));
    A(1,2) = A(1,2)+Qbar(1,2,ii)*(z(ii+1)-z(ii));
    A(1,3) = A(1,3)+Qbar(1,3,ii)*(z(ii+1)-z(ii));
    A(2,1) = A(2,1)+Qbar(2,1,ii)*(z(ii+1)-z(ii));
    A(2,2) = A(2,2)+Qbar(2,2,ii)*(z(ii+1)-z(ii));
    A(2,3) = A(2,3)+Qbar(2,3,ii)*(z(ii+1)-z(ii));
    A(3,1) = A(3,1)+Qbar(3,1,ii)*(z(ii+1)-z(ii));
    A(3,2) = A(3,2)+Qbar(3,2,ii)*(z(ii+1)-z(ii));
    A(3,3) = A(3,3)+Qbar(3,3,ii)*(z(ii+1)-z(ii));
end
%  Collect global, stiffness matrix and global compliance matrix
t = sum(t);
% Equivalent Modulus (ksi)
Ex = (A(1,1)*A(2,2)-A(1,2)^2)/t/A(2,2);
```

B5: get_ybar

```
function phi = get_ybar(ybar)

global inp
% Get Inputs
tbf = inp.tbf; Ebf = inp.Ebf; bbf = inp.bbf;ttf = inp.ttf;tw = inp.tw;
Ew = inp.Ew; Etf = inp.Etf;btf = inp.btf; fpc = inp.fpc; Ec = inp.Ec;
bc = inp.bc; D = inp.D;tc = inp.tc;k = inp.k; ds = inp.ds; Es = inp.Es;
fy = inp.fy; As = inp.As; gird=inp.gird; bsw = inp.bsw; bcurb = inp.bcurb;
dfrp = inp.dfrp;tcurb = inp.tcurb;

% Discretize section
n = 1000;
d = D;
if gird == 1 || gird == 5
    d = D+tcurb;
end
tn = d/n;
y = 0:tn:d;
F = 0;
% For each section devision
for ii = 1:length(y)
%     Calculate strain
    eps = -k*(y(ii)-ybar);
%     Calculate internal force contribution
    if y(ii) <= tbf
        f = Ebf*eps;
        A = bbf*tn;
    elseif y(ii) <= dfrp-ttf
        f = Ew*eps;
        A = tw*tn;
    elseif y(ii) <= dfrp
        f = Etf*eps;
        A = btf*tn;
    elseif y(ii) <= dfrp+0.5
        f = 0;
        A = 0;
    elseif y(ii) <= dfrp+0.5+tc
        f = Hognestad(eps,fpc,Ec);
        A = bc*tn;

    else
        if gird == 1
                f = Hognestad(eps,fpc,Ec);
                A = bsw*tn;
            elseif gird == 5
                f = Hognestad(eps,fpc,Ec);
                A = bcurb*tn;
            end
    end
    F = F+f*A;
end
% Calculate strain, stress in steel
```

```
epss = -k*(ds-ybar);
fs = Es*epss;
if abs(fs) > fy
    fs = fy*sign(fs);
end
% Calculate foce contribution from steel
Fs = fs*As;
% Find residual force in section
phi = abs(F+Fs);
```

B6: Hognestad

```
function fc = Hognestad(eps,fprimec,Ec)

% Record sign of strain, then take the absolute value
epssign = sign(eps);
eps = abs(eps);

% Determine concrete plastic strain and ultimate strain
epc = 1.8*fprimec/Ec;
eu = 0.003;
% Calculate concrete stress
if eps <= epc
    fc = fprimec*(2*eps/epc-(eps/epc)^2);
elseif eps <= eu
    fc = fprimec*(1-0.15*(eps-epc)/(eu-epc));
else
    fc = 0.95*fprimec;
end
fc = fc*epssign;
```

B7: getdisp

```
function [x,y] = getdisp

global msh L

% Create BVP mesh
msh = 0:L;
% Solve BVP
solinit = bvpinit(msh,@guessfun);
sol1 = bvp4c(@bvpfun,@bcfun,solinit);
v = sol1;
x = v.x;
y = v.y;
y = y(1,:);
```

B8: guessfun

```
function g = guessfun(x)
% Guess for form of displacement, rotation function
g = [x.^2;x.^3];
```

B10: bvpfun

```
function MoverEI = Mfun(x)

global EI L  girds

% Lane load Moment
MLane = 2*1.35*0.64/12*x/2*(L-x);
% Truck loads
P = 2*32*1.33*1.35;
% Calculate truck moment
if x <= L/2
    M1 = P*x/2;
else
    M1 = P*(L-x)/2;
end

if x <= L/2-14*12
    M2 = P*(L/2+14*12)*x/L;
else
    M2 = P*(L/2-14*12)*(L-x)/L;
end

if x <= L/2+14*12
    M3 = P*8/32*(L/2-14*12)*x/L;
else
    M3 = P*8/32*(L/2+14*12)*(L-x)/L;
end
Mtk = M1+M2+M3;
% Calculate total moment
if Mtk > 0.25*Mtk+MLane
    ML = Mtk;
else
    ML = 0.25*Mtk+MLane;
end
% Calculate curvature (M/EI)
M = ML;
MoverEI = M/EI/girds;
```

B11: bcfun

```
function res = bcfun(ya,yb)
% BVP boundary conditions
res = [ya(1) yb(1)];
```

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