Bi-Monthly Progress Report:
Project Number and Title: 1.6 Progressive fault identification and prognosis of railway tracks based on intelligent inference
Research Area: #1 Transportation infrastructure monitoring and assessment for enhanced life
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Reporting Period: ***04/1/2019 – 05/31/2019***
Date: ***05/31/2019***

Overview:

Provide overview and summary of activities performed during previous two months….

Owing to the electro-mechanical coupling of the piezoelectric transducer, the mechanical impedance of the host structure is related to the piezoelectric impedance. Therefore, the change of piezoelectric impedance signature with respect to that of the baseline healthy structure can indicate the damage occurrence. Because of the high-frequency nature of piezoelectric impedance, such method has been recognized to be quite sensitive to detect the small-size damage. An important element in the research of piezoelectric impedance method is to identify the location and severity of damage using measurement data. While the finite element analysis can provide an accurate method to predict the structural dynamic analysis, the damage identification based on the inverse analysis of such formulation is limited in practical application because of high computational cost and the usual under-determined nature of the problem. That is, the number of unknowns is generally very large so a usual matrix inversion may not lead to meaningful results.

Taking uncertainties into account, several probabilistic approaches i.e., perturbation method, Kriging predictor and random matrix theory have been suggested to characterize the underlying property of engineering structures. The Bayesian inference approach has shown certain advantages. For instance, it can specify the model parameters with prior information in the form of probability density function (PDF), which may be viewed as imposing soft physical constraints to enable a unique and stable solution. Moreover, this approach allows the computation of any type of statistics of the model parameters to be identified. The intent of this phase of the research is to develop a feasible and robust algorithm to conduct damage localization and identification based on piezoelectric impedance/admittance information. In particular, this proposed algorithm is built upon the Bayesian inference framework, which employs forward analysis-based approach instead of inversion-based identification procedures.

Provide context as to how these activities are helping achieve the overarching goal of the project…

The research activities conducted during this period lays down a solid foundation for the project in terms of fault detection and identification. While in the previous period the adaptive piezoelectric impedance sensor has shown promising aspects with high signal-to-noise ratio that reflects the fault feature, how to utilize the enhanced measurement becomes the next issue to be solved. In this project period, the Bayesian inference framework is established.

Describe any accomplishments achieved under the project goals…

To enhance the modeling accuracy, here we adopt the finite element method to carry out the modeling process, and the mathematical formulation is presented as follows. Without loss of generality, we consider a specific element. The constitutive relation for a piezoelectric material is

$$\boldsymbol{\sigma} = \mathbf{E}_p \boldsymbol{\varepsilon} + \mathbf{h} D, \qquad \mathbf{E} = \mathbf{h}^T \boldsymbol{\varepsilon} + \beta_{33} D \tag{1a ,b}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are, respectively, stress and strain vectors, $\mathbf{E}_p$ is the elastic matrix of the piezoelectric transducer, $\mathbf{h}$ accounts for the electro-mechanical coupling of the transducer which is defined in a specific form of $[-h_{31}, -h_{32}, 0, 0, 0, 0]^T$ in this research, D represents the electrical displacement, and $\mathbf{E}$ represents the electrical field. Using virtual work principle, we can have

$$\boldsymbol{\delta}_{(e)}^{*T} \mathbf{F}_{(e)} = \int_V \boldsymbol{\varepsilon}_{(e)}^{*T} \boldsymbol{\sigma}_{(e)}^T dV \tag{2}$$

which yields

$$\mathbf{F}_{(e)} = \int_V \mathbf{B}^T \mathbf{E}_p \mathbf{B} dV \boldsymbol{\delta}_{(e)}^* + \int_V \mathbf{B}^T \mathbf{h} D dV \tag{3}$$

Here we have *D=Q/A,* in which A is area of transducer. Following Equation (3), we have

$$\mathbf{F}_{(e)} = \mathbf{K}_{(e)} \boldsymbol{\delta}_{(e)}^* + \mathbf{k}_{12(e)} Q \tag{4}$$

where $\mathbf{K}_{(e)} = \int_V \mathbf{B}^T \mathbf{E}_p \mathbf{B} dV$, and $\mathbf{k}_{12(e)} = \dfrac{\int_V \mathbf{B}^T \mathbf{h} dV}{A}$. $\mathbf{B}$ is the stress-strain transformation matrix which is related to the shape function $\mathbf{N}$. Finally, we can derive the first governing equation of forced response as

$$\mathbf{M}_{(e)}\ddot{\boldsymbol{\delta}}_{(e)} + \mathbf{C}_{(e)}\dot{\boldsymbol{\delta}}_{(e)} + \mathbf{K}_{(e)}\boldsymbol{\delta}_{(e)} + \mathbf{k}_{12(e)}\mathbf{Q} = \mathbf{F}_{(e)} \tag{5}$$

where $\mathbf{M}_{(e)} = \int_V \mathbf{N}^T \rho \mathbf{BN} V$. Recall Equation (1) and perform the integration over the entire volume. We can also derive

$$\int_V \mathbf{E}_{(e)} dV = \int_V \mathbf{h}^T \boldsymbol{\varepsilon}_{(e)} dV + \int_V \beta_{33} D dV \tag{6}$$

As the transducer is connected with an inductor and subject to electrical resistance, we can finally obtain

$$L\ddot{Q} + R\dot{Q} + \mathbf{k}_{12(e)}^T \boldsymbol{\delta}_{(e)} + k_{22} Q = V \tag{7}$$

where $\mathbf{k}_{12(e)}^T = \dfrac{\int_V \mathbf{h}^T \mathbf{B} dV}{A}$, $k_{22} = \dfrac{\beta_{33} T_p}{A}$. L and R are respectively the inductance and resistance of circuitry.

When applying Bayesian theorem for structural model updating, the hypothesis $\boldsymbol{\theta}$ is interpreted as the vector of parameters that need to be identified. $\mathbf{D}$ denotes the measured signature, which in this study is the electrical admittance of piezoelectric transducer. M denotes modeling assumptions, reflecting the existing experience and knowledge. We then have

$$p(\boldsymbol{\theta} \mid \mathbf{D}, M) = \frac{p(\mathbf{D} \mid \boldsymbol{\theta}, M) p(\boldsymbol{\theta} \mid M)}{\int p(\mathbf{D} \mid \boldsymbol{\theta}, M) p(\boldsymbol{\theta} \mid M) d\boldsymbol{\theta}} \tag{8}$$

The prior distribution $p(\boldsymbol{\theta} \mid M)$ expresses the initial knowledge of concerned parameters, e.g., stiffness, mass, damage location. The choice of this distribution depends on how much information of the system is known. In this research, the parameters are only specified within certain range based on the prior information. Therefore, for the sake of simplicity, this term can be defined as a multivariate uniform distribution. The posterior distribution $p(\boldsymbol{\theta} \mid \mathbf{D}, M)$ indicates the updated knowledge of the parameters $\boldsymbol{\theta}$ conditional on the prior knowledge and measured admittance information. The likelihood function $p(\mathbf{D} \mid \boldsymbol{\theta}, M)$ is used to evaluate the agreement between the measurements and associated model output. Specifically, considering that uncertainties exist in real measurement, here we define the likelihood function as a multivariate normal distribution to conduct the screen of model output over $\boldsymbol{\theta}$ space.

$$p(\mathbf{D} \mid \boldsymbol{\theta}) = \frac{1}{\sqrt{(2\pi)^k \mid \Sigma \mid}} e^{-\frac{(\mathbf{D}-\mathbf{D}(\boldsymbol{\theta}))^T \Sigma^{-1} (\mathbf{D}-\mathbf{D}(\boldsymbol{\theta}))}{2}} \tag{9}$$

where $\mathbf{D}$ is a measured admittance vector, having length k , and $\mathbf{D}(\boldsymbol{\theta})$ is the model output parameterized by $\boldsymbol{\theta}$. Σ is the covariance matrix of $\mathbf{D}$.

Under this framework, the damage status of the structure monitored can be eventually identified. During this period of research, we have developed the finite element model of a benchmark plate structure as well as the Bayesian inference algorithm. The model is being thoroughly tested as compared with commercial software.

Describe any opportunities for training/professional development that have been provided…

This project currently involves on graduate student, Yixin Yao, to carry out the numerical and experimental investigations. This provides opportunity for training. The project progress is being communicated with industry collaborator, Sperry Rail Service, which provides another opportunity for training of state-of-the-art knowledge of active materials and advanced signal processing techniques for working professionals.

Describe any activities involving the dissemination of research results

In this phase of research, research results have been disseminated in the following occasions through tele-cons with Sperry Rail Service.

Participants and Collaborators:

Participants: Dr. Jiong Tang, PI, project lead; Yixin Yao, graduate student, research assistant.
Collaborator: Jan Kocur, Sperry Rail Service, providing technical assessment and industry insights.

Changes: N/A

Planned Activities:

The next phase of the research will continue the development of the fault identification algorithm, and explore the optimal tuning of adaptive sensor.